
Numerical optimisation methods applied to the concurrent problem of vehicle ride and handling

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Abstract: This text presents a new proposal for an overall ride and handling metric to be used to optimise vehicle performance through concurrent simulation models. The proposed metric has the intention to cover simultaneously the most relevant aspects of a passenger vehicle dynamic behaviour. The individual models for ride and handling have been validated against physical measurements in order to assure a reliable output. A numerical optimisation method is used to simultaneously evaluate the complete vehicle dynamic performance related to the single overall metric. The results show an effective way to improve the vehicle performance based on this new proposed metric, with gains in terms of reduction in development time and cost for new projects.

Keywords: ride; handling; vehicle dynamics; simulation; optimisation.

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1 Introduction

Current literature presents a vast number of models and simulation tools that allow the development engineer to predict the vehicle ride and handling behaviour with very good accuracy. Some of these models are described analytically and covered in the traditional literature for vehicle dynamics, like Milliken and Milliken (1995), Wong (2001) and Pacejka (2002). Other analytical tools are developed with specific purposes, like Gordon et al. (2012) using a quarter-car model to study the influence of the road surface for ride tuning and Talukdar et al. (2012) comparing different simplified models for ride characterisation. These models are simple, therefore extremely efficient in computational terms, making them natural choices to apply numerical optimisation processes. The main drawback in adopting these analytical models from traditional literature is that there is very few data available related to their accuracy against real vehicle measurements. This fact demands additional work to understand the level of model detail necessary to adequately capture the quantities of interest.

Another possibility for the design engineer is to consider a detailed multibody model, making use of commercial multibody software packages, as it has been done using ADAMS[®] in works by Rongshan et al. (2010), Wu et al. (2009), Vilela (2001) and Prado et al. (2001) – this is currently a widely disseminated approach. By adopting these more complex models the engineer can get very accurate results for the vehicle dynamics response – Rill (2006) and Adamski et al. (1999) describe in more detail how some of these multibody models work, showing the benefits of the flexibility they bring to the design engineer. The issue with this approach is that, as the multibody model gets details in the vehicle construction representation (a common multibody model easily contains more than 100 degrees of freedom), it also becomes less efficient in computational running time. Even with a constantly increasing computational capability available to the design engineer, this non-optimal efficiency might become a bottleneck for numerical optimisation procedures as it demands high number of iterations to get to an optimum design. Besides, these complex models can make it less intuitive for the engineer the understanding of the physics behaviour of the phenomena being studied. These characteristics from the detailed multibody models make the selection of proper tuning variables for the optimisation process more difficult.

The literature for simultaneous analysis of ride and handling is still limited, except for some works using these commercial packages like Rongshan et al. (2010), Yang and Gander (2010) and Wu et al. (2009). Others authors are focused on active control strategies like Liu and Ya (2012), Chen et al. (2011), Rengaraj and Crolla (2011) and Nikzad and Naraghi (2001). Most of these works still propose generic metrics to describe the ride and handling compromise, like Johnston et al. (2010) did by using vertical response spectral profiles for ride quantification and the coefficient of variation of the loads as experienced by the contact patch for handling quantification (what was defined as ‘grip’ in that work), but these metrics are more difficult to be directly correlated with physical measurements as the authors acknowledge.

This work proposes concurrent use of analytical tools for ride and handling simultaneously through metrics that are current practice for automotive vehicles’ development. The handling models used were developed by Vilela and Barbosa (2011a, 2011b) and the numerical tools for ride comfort were developed by Vilela and Gueler (2003) and Vilela and Tamai (2003a, 2003b). These tools present two common key characteristics that make them very suitable for a numerical optimisation process: they

are extremely efficient computationally and, at the same time, they present accurate and consistent results with respect to the physical phenomena they intend to reproduce. This aspect has been confirmed by means of direct correlation with experimental measurements. It is also applied the response surface method (RSM) [as presented by Myers and Montgomery (2002) and Myers et al. (2004)] to the concurrent model proposed in order to understand the potential benefits of this tool for the automotive industry.

2 Objective metrics

The main roadblock that needs to be surpassed to consider a numerical optimisation process for any given problem is to obtain quantitative metrics that represent well the phenomenon studied. In the case of the ride and handling optimisation, these metrics need to meet the following criteria:

- *Be objective*: it is mandatory to consider objective metrics to be used with the optimisation process during the suspension set-up phases, even though ride and handling are areas where a good subjective impression by the final user is the ultimate goal during the vehicle development. These objective metrics must also be quantifiable.
- *Present good objective-subjective correlation*: the objective metrics considered must capture adequately the subjective impressions from the human drivers. At the same time however, these metrics must be independent of human judgement (i.e., it should be possible to measure/calculate the final value of the metric directly from physical variables like accelerations, displacements and forces) in order to make them effective for an automatic optimisation process.
- *Present good numerical-physical correlation*: for a complete optimisation process, the metrics considered must be calculated by computer simulation (analytical formulation or interactive analysis) and the results obtained by these numerical formulations must be consistent with the experimental results measured in physical vehicles.

In the sequence the various metrics considered for the numerical optimisation study are presented.

2.1 Ride comfort metrics

The ride comfort metrics considered are related with the vehicle's capacity to filter vertical road inputs and are described in the sequence. A detailed work with the proving ground expert drivers has been performed in order to determine the kind of road input where each one of the metrics defined is more evident and, as much as possible, isolated from the others. The metrics herein shown were initially described by Vilela et al. (2002).

- *Harshness*: capacity of the vehicle suspension to filter high frequency irregularities with low amplitude road inputs. *Road input*: Belgium blocks track.
- *Absorption capability*: capacity of the vehicle suspension in absorbing the impact with medium size obstacles on the road surface, small size wavelength perturbation

like cats' eyes or little parts of stone. *Road input*: one side of the vehicle upon cats' eyes (50 mm high) and the other on a smooth paved road.

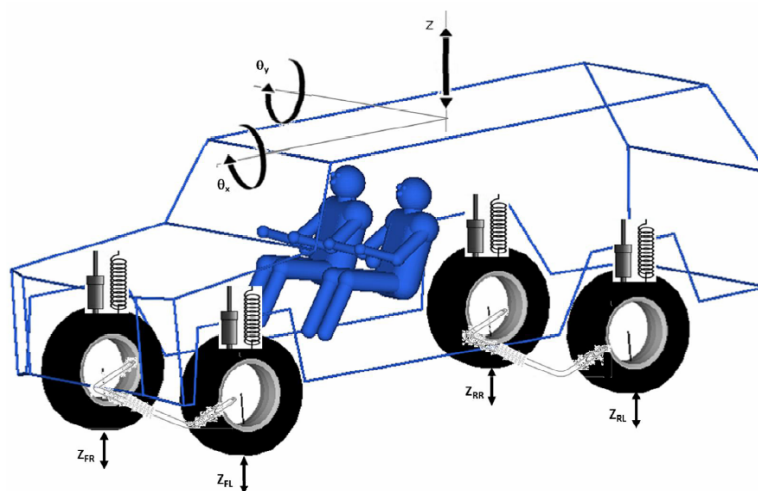
- *Jounce bumper*: evaluates the vehicle behaviour when passing through pot holes, concerning the bump impact felt by the driver (medium size wavelength input). This metric uses the same name of the suspension component in order to keep the common terminology used by the proving ground evaluators, which is related to the fact that the typical pot hole inputs usually excite the jounce bumper component. *Road input*: series of pot holes in both sides of the track.
- *Ride balance*: evaluates the vehicle behaviour when passing through cross ditches or similar obstacles on the road (long wavelength roughness), concerning the pitch stability of the vehicle. *Road input*: cross ditch followed by a flat road.

Finally, the dynamic variables were correlated to each ride comfort parameter. The following parameters were taken into consideration:

- vertical acceleration at the driver position
- front and rear forces for the spring, shock absorbers and bumpers
- body pitch and roll accelerations.

The variables above were calculated by means of a simplified multibody model illustrated in Figure 1 and with general state equations described in equations (1) to (3). These variables were then combined by means of a weight matrix, resulting in values that were correlated with the subjective grades given by the expert drivers at the experimental proving ground. This methodology was developed to correlate the dynamic ride comfort variables from the simulation with the subjective grades from the expert drivers. It presented good correlation, allowing its usage with the upcoming numerical optimisation process – more details about the correlation results are presented in Vilela et al. (2002), Vilela and Gueler (2003) and Vilela and Tamai (2003a, 2003b).

Figure 1 Sketch of the vertical vehicle model for ride comfort metrics calculation (see online version for colours)



$$\Delta x[i] = \sum_{j=1}^{N_M + N_I + N_{EX}} (Inf[i, j] \cdot x[j]) \quad (1)$$

$$\Delta \dot{x}[i] = \sum_{j=1}^{N_M + N_I + N_{EX}} (Inf[i, j] \cdot \dot{x}[j]) \quad (2)$$

$$F[k] = \sum_{l=1}^{N_{Link}} [-Inf[l, k] \cdot (F_{Stiff}[l] + F_{Damp}[l])] \quad (3)$$

2.2 Handling metrics

The handling metrics are related to the capacity of the vehicle to generate tangential contact loads with minimum variation of vertical contact loads. Based on common development practice, the following metrics were considered:

- roll gradient
- understeer gradient
- steering sensitivity
- lateral acceleration response
- roll response.

2.2.1 Roll gradient metric

The roll gradient is defined as the derivative of the vehicle body roll angle with respect to the lateral acceleration acting at its centre of gravity (CG), as indicated in Figure 2 (curve point highlighted by red line represents the lateral acceleration level where this metric is being calculated for the current work). The curve itself is generated by a series of points where the steady-state condition is observed. This value can be experimentally measured through a constant radius circular manoeuvre with small steps of increase in the longitudinal velocity (and therefore the lateral acceleration), keeping as close as possible of a steady-state condition.

An analytical model, described by equation (4), has been developed for this metric and correlated against experimental results as shown in Figure 3. Experimental data was acquired for a passenger vehicle with instrumented steering wheel angle, longitudinal velocity, lateral acceleration (accelerometers at vehicle's CG position) and roll angle with respect to the ground. The manoeuvre performed for the data acquisition was a slowly increasing longitudinal velocity over a constant radius and it has been repeated three times to assure that measurement results were consistent. The details of the measurements and correlation results are presented by Vilela and Barbosa (2011a).

$$K_{roll} = \frac{\partial \theta}{\partial a_L} = \frac{MgH_r}{K_T} \quad (4)$$

Figure 2 Roll gradient definition (see online version for colours)

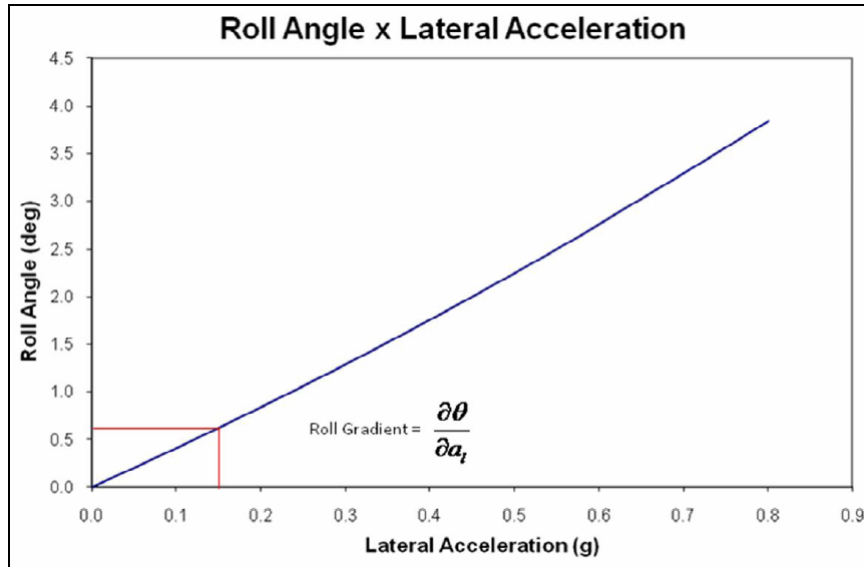
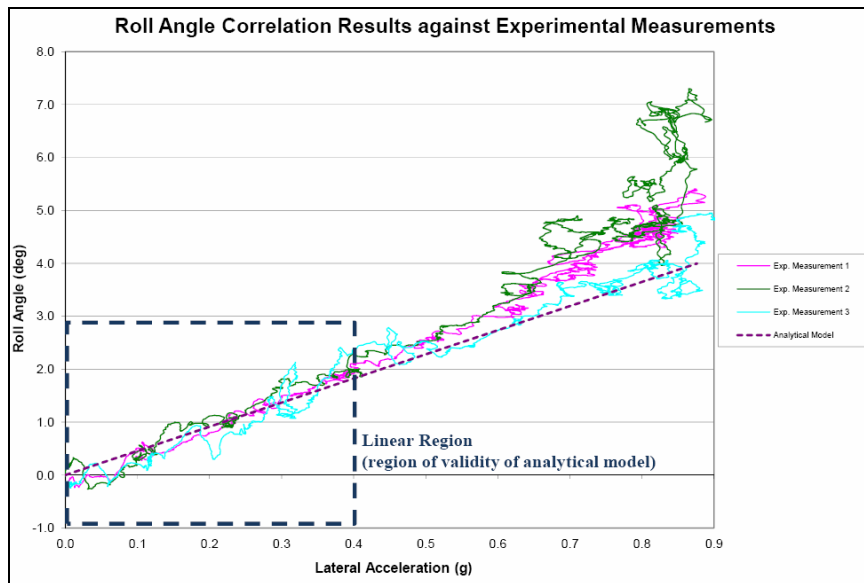
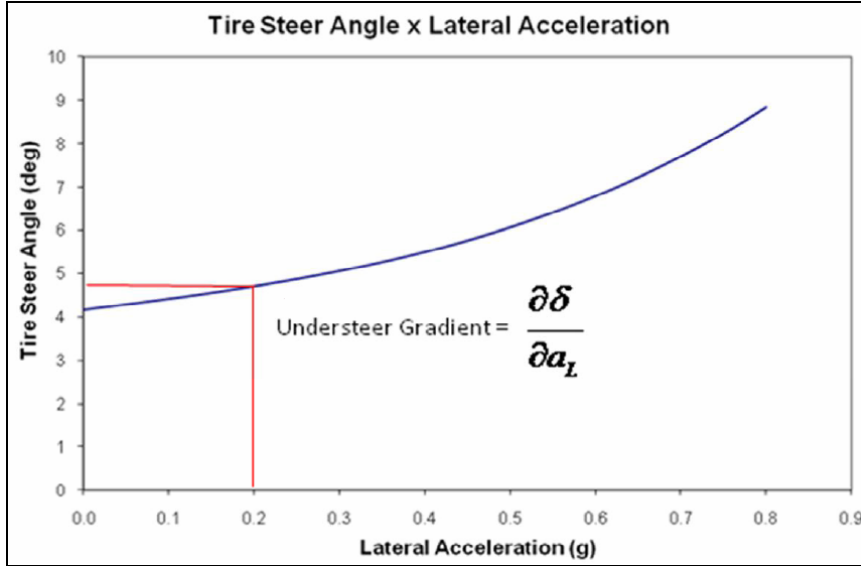


Figure 3 Roll gradient correlation results against experimental measurements (see online version for colours)



2.2.2 Understeer gradient metric

The understeer gradient is defined as the derivative of the front tyres average steer angle with respect to the lateral acceleration imposed to the vehicle at its centre of gravity in a steady-state condition, as indicated in Figure 4.

Figure 4 Understeer gradient definition (see online version for colours)

This parameter evaluates the tendency of the vehicle to be understeer when in a steady-state curve manoeuvre (understeer gradient $> 0 \rightarrow$ vehicle demands higher steering angles to keep the same curve radius at higher speeds) or oversteer (understeer gradient $< 0 \rightarrow$ vehicle demands lower steering angles to keep the same curve radius at higher speeds). The vehicle is said to be neutral when the steering angle to keep a curve trajectory is dependant only on the curve radius and not on the vehicle speed (understeer gradient null).

An analytical model capable of reproducing the experimental results with good accuracy was developed by Vilela and Barbosa (2011a). The development starts from a simple bicycle model that was gradually implemented with the effects of tyre self-align torque, lateral load transfer, vehicle's suspension and steering system compliances and suspension kinematic variation with vertical suspension travel. This analytical model is described by equations (5) to (7) and its development and correlation against experimental results is detailed by Vilela and Barbosa (2011a).

$$K = \frac{\partial \delta}{\partial a_L} = \left(\frac{c}{C'_{\alpha_f}} - \frac{b}{C'_{\alpha_r}} \right) \frac{M}{2L} \quad (5)$$

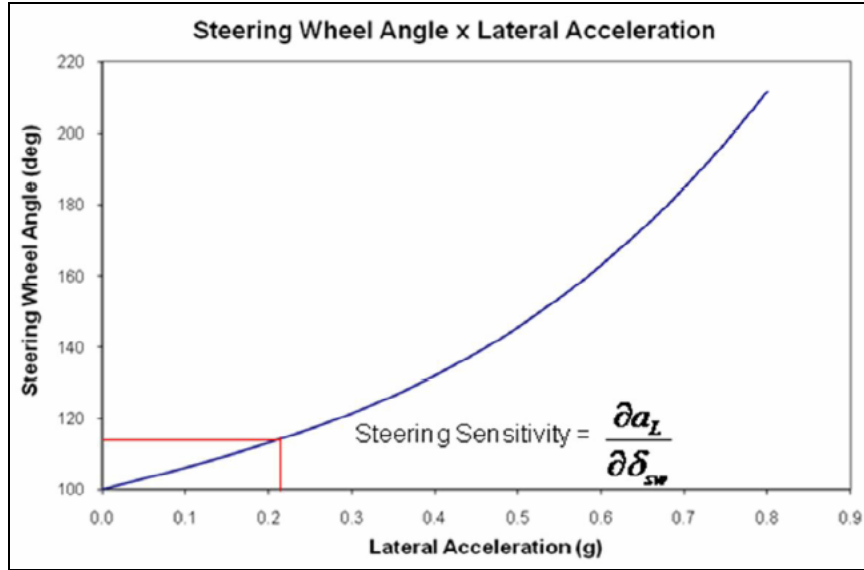
$$C'_{\alpha_f} = \frac{C_{\alpha_f,lt}c}{B_f B_{f,rs}c'} \quad (6)$$

$$C'_{\alpha_r} = \frac{C_{\alpha_r,lt}b}{B_r B_{r,rs}b'} \quad (7)$$

2.2.3 Steering sensitivity metric

The steering sensitivity is defined as the derivative of the lateral acceleration with respect to the steering wheel angle imposed to the vehicle's centre of gravity, as indicated in Figure 5.

Figure 5 Steering sensitivity definition (see online version for colours)



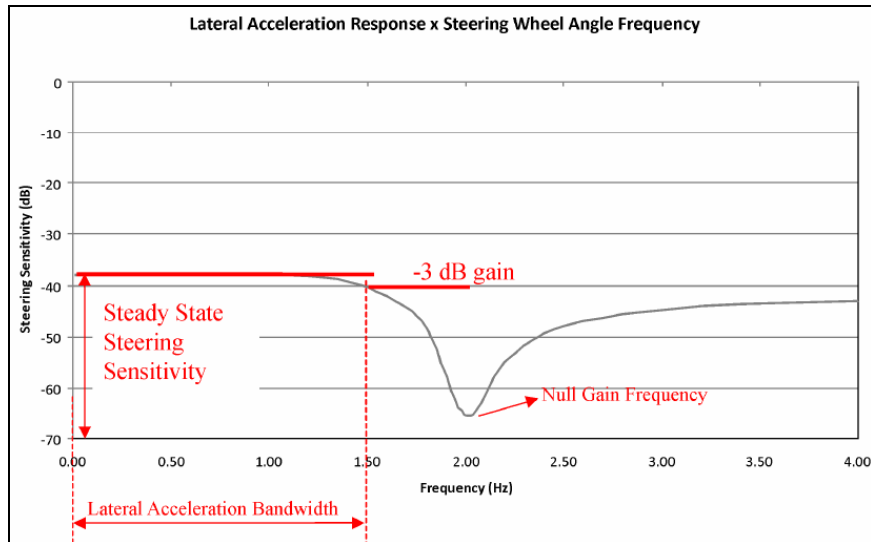
This parameter evaluates the responsiveness of the vehicle with respect to the driver inputs at the steering wheel. Low steering sensitivity values disclose a subjective feeling to the driver of a slow response or lack of response from the vehicle. High values instead are associated with very fast vehicle response that is more difficult to control. In this case, a small disturbance in the steering wheel produces a reasonable amount of lateral acceleration, changing significantly the vehicle trajectory. The steering sensitivity is closely related to the understeer gradient, being inversely proportional to that metric and to the overall steering ratio of the vehicle. The reason that makes it relevant to consider this metric independently is that many projects are limited to use the same steering system for a wide range of vehicles. This fact makes the compromise between understeer gradient and steering sensitivity more difficult to be achieved. Besides that, there is also a compromise between this metric and the steering effort, as the steering sensitivity is inversely proportional to the overall steering ratio of the vehicle, what is especially critical for non-assisted (manual) steering systems. This analytical model is summarised by equation (8) and its development and correlation against experimental results is also detailed by Vilela and Barbosa (2011a).

$$K_s = \frac{\partial a_L}{\partial \delta_{vol}} = \frac{\partial a_L}{\partial \delta} \frac{\partial \delta}{\partial \delta_{vol}} = \frac{1}{K} \frac{1}{r_{dir}} \quad (8)$$

2.2.4 Lateral acceleration response metrics for periodic excitation

The lateral acceleration response of the vehicle with respect to the excitation frequency of the steering wheel (harmonic response to a sinusoidal excitation type) presents a decreasing behaviour at the beginning of the response curve. Eventually it achieves a minimum response value for a specific frequency. This is called *null gain frequency*, as the gain value at this frequency is very close to zero. Another metric that can be taken from this response is the so-called *lateral acceleration bandwidth* – it is defined as the frequency value where a reduction in the lateral acceleration response is noticed by most users. The reduction in 3 dB gain for this metric proposed by Kunkel and Leffert (1998) is adopted in this work. Figure 6 shows these concepts at the lateral acceleration response graphic.

Figure 6 Lateral acceleration bandwidth and null gain frequency definition (see online version for colours)



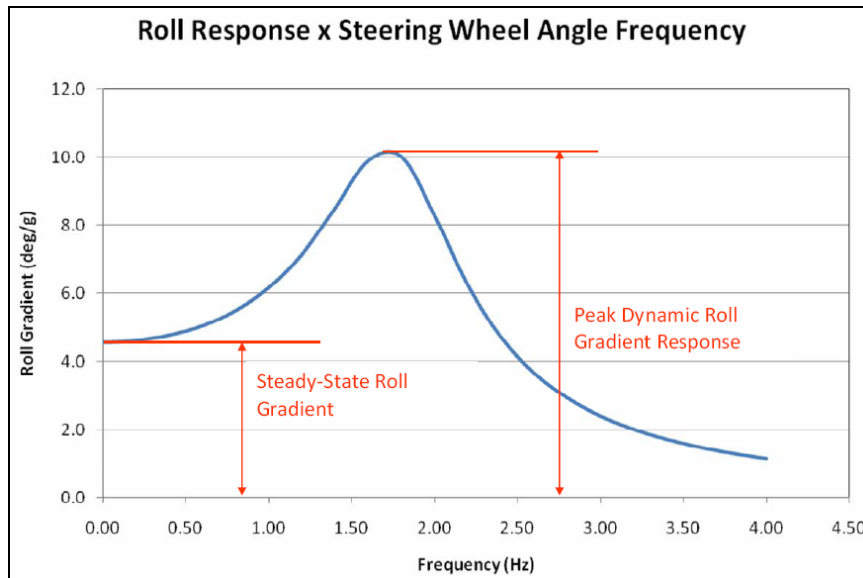
Vilela and Barbosa (2011b) detail the development and correlation of an analytical model capable of replicating the behaviour of the lateral acceleration response as presented in Figure 6, with results compared against a detailed multibody model and a simpler analytical model proposed by Pacejka (2002). This model is summarised by equation (9).

$$\begin{aligned}
 & \left[\frac{\left(M + \frac{2}{V_x^2} (C_{\alpha_f} b - C_{ar} c) \right)}{2(C_{\alpha_f} + C_{ar})} J_z \right] \ddot{a}_L + \left[\frac{1}{V_x} J_z \right] \dot{a}_L \\
 & + \left[\frac{2}{V_x^2} (b^2 C_{\alpha_f} + c^2 C_{ar}) - \frac{\left(M + \frac{2}{V_x^2} (C_{\alpha_f} b - C_{ar} c) \right) (2(b C_{\alpha_f} - c C_{ar}))}{2(C_{\alpha_f} + C_{ar})} \right] a_L \quad (9) \\
 & = \left[2C_{\alpha_f} \left(b - \frac{2(b C_{\alpha_f} - c C_{ar}) + J_z \omega^2}{2(C_{\alpha_f} + C_{ar})} \right) \right] \delta
 \end{aligned}$$

2.2.5 Roll response metric for periodic excitation

The vehicle body roll angle behaviour with respect to the excitation frequency of the steering wheel (harmonic response to a sinusoidal excitation type) presents a typical second order system behaviour as depicted in Figure 7. The main metric that can be taken here is the dimensional measure defined by the peak roll gradient response divided by the response at steady-state condition ($\omega \rightarrow 0$), which is called *roll gradient peak/steady-state ratio*. This metric indicates how much the vehicle roll response varies with respect to the steering wheel excitation frequency. Lower values (close to 1.0) are better perceived by the users, as it reflects a more homogeneous response, independent of the steering wheel excitation frequency. Higher values can bring a strong non-linearity feeling to the users with respect to the roll response and extreme cases might affect the safety of the vehicle with respect to rollover. This metric is strongly affected by the suspension damping properties. Usually there is a compromise between the roll dynamic response and the vehicle ride comfort.

Figure 7 Roll gradient peak/steady-state ratio definition (see online version for colours)



Vilela (2010) developed an analytical model, described by equation (10), that is capable of replicating the behaviour of the roll angle response as presented in Figure 7 and the correlation results against a detailed multibody model is detailed in that work.

$$J_x \ddot{\theta} - C_T \dot{\theta} - K_T \theta = -MH_r a_L \quad (10)$$

All handling metrics considered in this work were proposed by Vilela and Barbosa (2011a, 2011b). Table 1 presents the correlation results of the proposed numerical models for a passenger vehicle, with the correlation level calculated as per definition from equation (11). The steady-state metrics (roll gradient, understeer gradient and steering sensitivity) reference values were experimentally measured in the vehicle performing a constant radius manoeuvre. The frequency response metrics reference values were

obtained from a detailed multibody model of the same vehicle. Correlation is defined as 100% when proposed analytical model results matches reference results from experimental measurements and/or detailed multibody model.

$$\text{Correlation level} = \frac{\min(\text{Experimental result}, \text{Proposed model result})}{\max(\text{Experimental result}, \text{Proposed model result})} \quad (11)$$

Table 1 Handling metrics correlation results

<i>Metric</i>	<i>Unit</i>	<i>Reference value</i>		<i>Proposed model result</i>	<i>Correlation level (%)</i>
		<i>Experimental result</i>	<i>Detailed multibody model result</i>		
Roll gradient	deg/g	4.93	4.59	4.56	92.5%
Understeer gradient	deg/g	3.85	3.54	3.92	98.2%
Steering sensitivity	g/100 deg SWA	1.55	1.66	1.50	96.8%
Lateral acceleration bandwidth	Hz	N/A	1.35	1.53	88.2%
Null gain frequency	Hz	N/A	2.09	2.04	97.6%
Roll gradient peak/steady-state ratio	-	N/A	2.24	2.24	100.0%

2.3 Overall ride and handling metric

The application of the numerical optimisation tools herein considered demands a single combined metric. The intention is to propose a combined metric to optimise ride and handling metrics simultaneously.

Figure 8 Nominal-the-better metrics normalisation (see online version for colours)

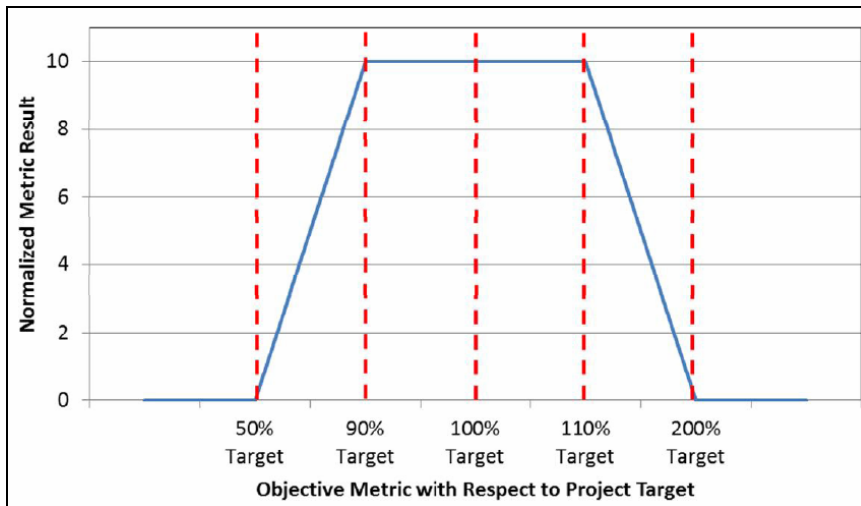


Figure 9 Higher-the-better metrics normalisation (see online version for colours)

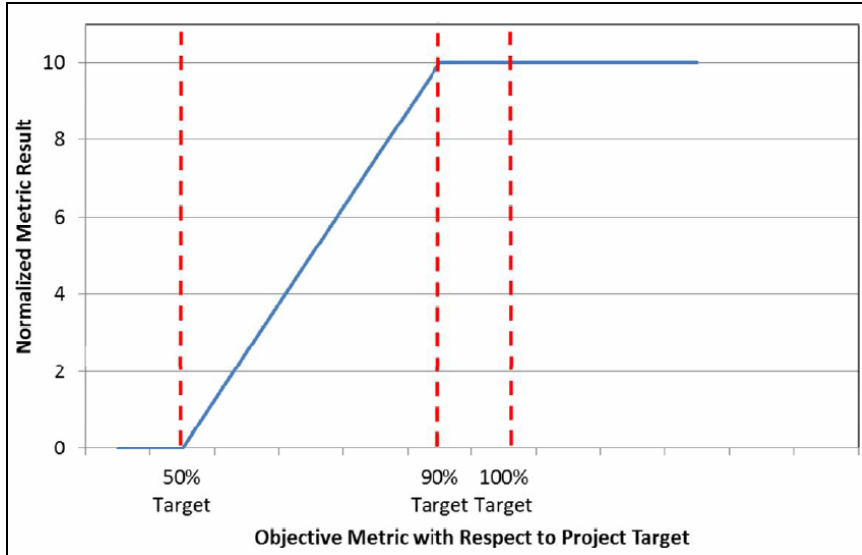
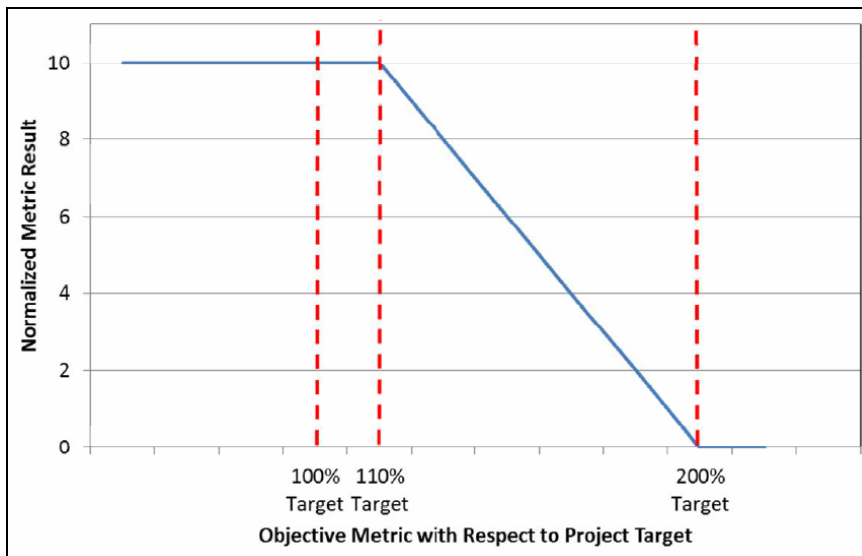


Figure 10 Lower-the-better metrics normalisation (see online version for colours)



The ride comfort metrics previously described already present a ‘higher-the-better’ characteristic (i.e., higher values of the metric imply in a better behaviour of the vehicle with respect to that metric) and are already normalised to produce results between 0 and 10. On the other hand, the handling metrics described demand some manipulation for the final metric combination. A specific vehicle project has clearly defined targets for each handling metric considered. These metrics present either a nominal-the-better characteristic (understeer gradient and steering sensitivity), a higher-the-better characteristic (lateral acceleration bandwidth) or a lower-the-better characteristic (roll

gradient and roll gradient peak/steady state ratio). Mathematical manipulation through linear functions can be then applied to normalise the result to a value between 0 and 10, as described in Figures 8 to 10 in the sequence.

The overall handling metric can be then defined as a weighted average of the individual metrics, with each individual weight factor defined based on the vehicle project specific goals, which are dependent on its application (a vehicle can be sportive, family-oriented or off-road for example) and expected user behaviour (one vehicle might be designed to younger drivers, while another might be oriented to families and/or senior public). In this sense, with weight factors defined as p_1 until p_5 :

$$f(\text{handling}) = \frac{p_1 f(K_{roll}) + p_2 f(K) + p_3 f(K_s) + p_4 f(R_{roll}) + p_5 f(\omega_{plane})}{\sum_{i=1}^5 p_i} \quad (12)$$

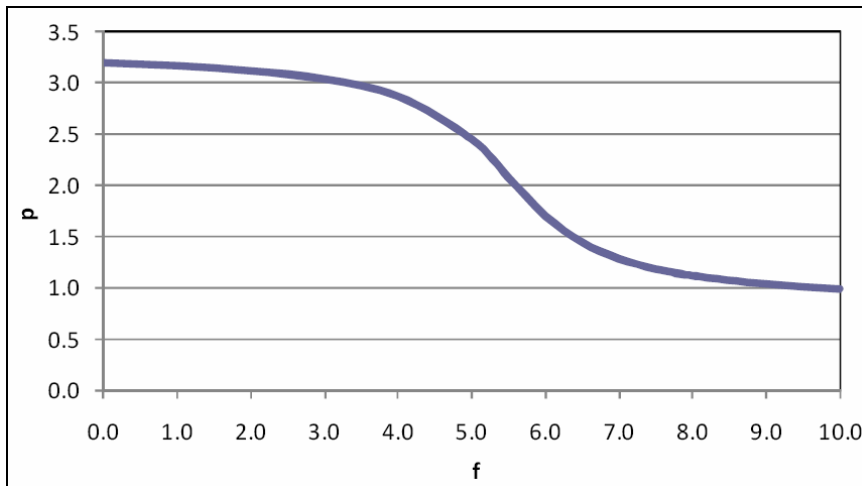
Finally, the combined ride and handling metric can be also defined as a weighted sum between $f(\text{ride})$ and $f(\text{handling})$. Once more, the weight factors must be defined according to the specific project needs.

$$f(\text{optimisation}) = \frac{p_{ride} f(\text{ride}) + p_{handling} f(\text{handling})}{p_{ride} + p_{handling}} \quad (13)$$

The metric defined by equation (13) is also normalised between 0 and 10 and is suitable for the numerical optimisation procedures proposed.

Besides these properties, it is very important that the numerical optimisation does not seek for solutions that emphasise too much a subset of the metrics despite bad results for others. Among available empirical adjustments, the hyperbolic function described in Figure 11 has been chosen. This additional factor increases the relative weight of the metrics with lower values, forcing thus the optimisation procedure to seek for more balanced results among the various metrics.

Figure 11 Additional weight factor proposed (see online version for colours)



3 Application of numerical optimisation

As the ride comfort model is an interactive multibody model, a direct regression for the optimisation study is not possible. For this reason, the RSM as described by Myers and Montgomery (2002) and considered by Vilela and Tamai (2005) is explored in this work in order to understand the effectiveness of the proposed metric for numerical optimisation purposes. The objective of the optimisation is to maximise the function defined by equation (13), what is equivalent to minimise the same function with opposite sign as described in equation (14), considering the variables x within the limits of the considered admissible set A . The variables and limits considered for this study are presented in Table 2. Specifically for the tyres, the various inflation pressures were considered by adopting the equivalent radial stiffness (used in the ride and roll models) and cornering stiffness (lateral force vs. slip and align torque vs. slip coefficients for the handling models).

$$\text{Min}_{x \in A}(-f(x)) \quad (14)$$

Table 2 Optimisation variables and limits considered

<i>Variable</i>	<i>Minimum</i>	<i>Maximum</i>	<i>Unity</i>
x_1 – rear axle antiroll bar stiffness	0 (no bar)	10	N/mm
x_2 – front spring stiffness	18	22	N/mm
x_3 – rear spring stiffness	18	22	N/mm
x_4 – front antiroll bar diameter	18	22	mm
x_5 – front tyre inflation pressure	26	34	psi
x_6 – rear tyre inflation pressure	26	34	psi
x_7 – front shock damping	–20% over nominal	+20% over nominal	N/(m/s)
x_8 – rear shock damping	–20% over nominal	+20% over nominal	N/(m/s)

The noise factor considered was the vehicle ballast condition, with the ride and handling metrics calculated at ballast = kerb + two front passengers and at ballast = gross vehicle mass (GVM). Table 3 shows the handling metric targets that were considered along with the normalisation presented in Figures 8, 9 and 10.

Table 3 Targets for handling metrics

<i>Metric</i>	<i>Unity</i>	<i>Criteria</i>
Roll gradient	deg/g	5.0
Understeer gradient	deg/g	2.5
Steering sensitivity	g/100 deg SWA	1.5
Roll gradient peak/steady-state ratio	-	1.5
Lateral acceleration bandwidth	Hz	1.2

For this study, the project-related weight factors are all kept equal to unity, so as to make this study a completely balanced ride and handling optimisation with respect to the metrics considered. The additional weight factors presented in Figure 11 are maintained as defined (i.e., dependant on the metric result) so as to help the numeric optimisation routine to find a balanced optimum point.

A fractional factorial matrix as detailed by Vilela (2010) has been considered for this work. Considering the backward elimination method it is possible to obtain the most significant regressors as shown in Table 4.

Table 4 RSM most significant regressors

<i>Factor</i>	<i>Partial regression coefficient</i>
Intersection	+7.068
Ballast condition (noise factor)	-0.501
Front shock damping	-0.365
Front antiroll bar stiffness	+0.171
Front tyre inflation pressure	-0.152
Interaction between ballast condition and rear tyre inflation pressure	+0.107
Rear axle torsional stiffness	+0.094
Interaction between ballast condition and front antiroll bar stiffness	+0.085
Interaction between rear axle torsional stiffness and front antiroll bar stiffness	-0.076
Interaction between ballast condition and front shock damping	+0.066
Interaction between ballast condition and rear axle torsional stiffness	+0.046
Rear spring stiffness	+0.041
Interaction between rear spring stiffness and front antiroll bar stiffness	-0.040

One possible way to quantify the accuracy of the RSM regression models is to compare their results directly with the original results from the fractional factorial matrix, as shown in Table 5.

Table 5 RSM models accuracy check

	<i>RSM regressors</i>
Average difference between fractional factorial results and RSM model	0.08
Maximum positive difference (RSM model > original result)	+0.37
Maximum negative difference (RSM model < original result)	-0.24

Table 6 Ballast condition variability study using RSM model

<i>Factor</i>	<i>Initial configuration</i>	<i>Minimum variability</i>	<i>Maximum metric value</i>
	<i>Metric = 7.27 Var = 0.43</i>	<i>Metric = 7.81 Var = 0.15</i>	<i>Metric = 7.88 Var = 0.29</i>
Rear axle torsional stiffness	0.00	+1.00	+1.00
Front antiroll bar stiffness	0.00	+1.00	+1.00
Front tyre inflation pressure	0.00	-1.00	-1.00
Rear tyre inflation pressure	0.00	+1.00	-0.35
Front shock damping	0.00	-1.00	-1.00

As the RSM obtains simple linear equations with the regressors, it is possible to use these equations with common linear quadratic optimisation routines readily available in

software packages like MatLab® and SciLab®, allowing the design engineer to understand how the global ride and handling optimisation metric varies as function of the optimisation variables and the ballast condition (variability as function of noise considered). Table 6 presents some results considering this aspect.

The RSM results bring a series of extremely useful information to the design engineer. As example, from the partial regression coefficients from Table 3, the following observations can be done:

- Within the range of this study, the vehicle ballast condition presents the highest regressor. That means that the noise produced by the ballast condition has more influence in the global ride and handling metric than any of the optimisation variables alone (shocks, springs, tyres, etc.). The fact that the coefficient is negative means that the metric is reduced when the variable value increases, i.e., the global ride and handling metric considered is degraded as the vehicle ballast increases, what is aligned with passenger vehicles usual response.
- The interaction between the ballast and the rear tyre inflation pressure is positive: that means that for higher ballasts (positive values for variable), it is interesting to increase the rear tyre inflation pressure to improve the vehicle ride and handling. This conclusion is also very much aligned with the common practice to consider different tyre inflation pressures for the tyres with different ballast conditions (especially true for the rear tyre).
- The interaction between the vehicle ballast and the front antiroll bar torsional stiffness is also positive: that means that the heavier ballasted vehicle will perform better with a stiffer front bar for the ride and handling global metric considered. This kind of information can help the design engineer to idealise different solutions for the project – in this example, it might be feasible to study an antiroll bar whose stiffness increases as function of the ballast or, alternatively, a front spring with progressive rate that will end up increasing the effective roll stiffness of the front axle as the ballast increases.

Besides these points, a direct conclusion can be obtained by a comparison of the partial regression coefficients' magnitude, what can help the design teams to put more effort in the components/variables that affect most the results.

The goal of improving the initial design has been achieved: in the example described here this improvement was in the range of 8% (around 0.6 points over initial 7.3 in a scale from 0 to 10) only with the application of the numerical optimisation process over regular suspension tuning components (springs, tyres, shock absorbers and antiroll bars), meaning that this gain was obtained at no extra cost to the product.

4 Conclusions

A new unified concurrent ride and handling metric is proposed to be used with numerical tools to optimise vehicle suspension design. The models developed include the relevant aspects to represent the physical phenomena involved, having at the same time a simple structure and providing good numerical efficiency. The models were validated against experimental values and more detailed and complex multibody models.

The RSM has been applied to this new unified ride and handling metric and the goal of improving the initial design has been achieved. Along with the application of the RSM optimisation process with the proposed model, significant insightful information about the interactions among the variables was obtained.

These points indicate that the proposed model and the optimisation methodology aid the design engineer and bring a significant contribution to the automotive companies. Their application can ultimately accelerate the development process through the usage of numerical methods over traditional hardware work in the initial phases of their projects, reducing associated costs and timing demands at the same time.

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Appendix*Definitions, acronyms and abbreviations*

a_L	Lateral acceleration
b / c	Distance between CG and front/rear axle
b' / c'	Adjusted distance between CG and front/rear axle considering tire self-align torque
B_f, B_r	Front/rear suspension/steering system compliance term
$B_{f,rs}, B_{r,rs}$	Front/rear roll steer term
$C_{\alpha,lt}, C_{\alpha,rt}$	Front/rear tire cornering stiffness with lateral load transfer effect
$C'_{\alpha,f}, C'_{\alpha,r}$	Equivalent front/rear tire cornering stiffness
C_T	Vehicle total roll damping
$F[k]$	Total force acting on link system $[k]$
$F_{Stiff}[l]$	Force acting on link system $[k]$ due to stiffness components
$F_{Damp}[l]$	Force acting on link system $[k]$ due to damping components
g	Gravity acceleration (9.81 m/s^2)
H_r	Effective roll arm
$Inf[i, j]$	Influence matrix component relating link system $[i]$ to link system $[j]$
J_x	Vehicle roll moment of inertia (x axis)
J_z	Vehicle yaw moment of inertia (z axis)
K	Understeer gradient
K_{roll}	Roll gradient
K_s	Steering sensitivity
K_T	Vehicle total roll stiffness
L	Wheelbase
M	Vehicle mass
r_{dir}	Steering ratio
V_x	Vehicle longitudinal velocity
$x[i]$	Position of link system $[i]$ in ride multibody model
$\dot{x}[j]$	Velocity of link system $[i]$ in ride multibody model
δ	Front wheel steer angle
δ_{vol}	Steering wheel angle
θ	Vehicle roll angle
ω	Frequency of steering wheel excitation
