

Vehicle Dynamic Safety in Measured Rough Pavement

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Abstract: Dynamic vehicle behavior is used to identify safe traffic speed limits. The proposed methodology is based on the vehicle vertical wheel contact force response excited by measured pavement irregularities on the frequency domain. A quarter-car model is used to identify vehicle dynamic behavior. The vertical elevation of an unpaved road surface has been measured. The roughness spectral density is quantified as ISO Level C. Calculations for the vehicle inertance function were derived by using the vertical contact force transfer function weighed by the pavement spectral density roughness function in the frequency domain. The statistical contact load variation is obtained from the vehicle inertance density function integration. The vehicle safety behavior concept is based on its handling ability properties. The ability to generate tangential forces on the wheel/road contact interface is the key to vehicle handling. This ability is related to tire/pavement contact forces. A contribution to establish a traffic safety speed limit is obtained from the likelihood of the loss of driveability. The results show that at speeds faster than 25 km/h the likelihood of tire contact loss is possible when traveling on the measured road type. DOI: 10.1061/(ASCE)TE.1943-5436.0000216. © 2011 American Society of Civil Engineers.

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Introduction

It is clear for the vehicle driver that faster traveling speeds result in less travel time. It is also proven that safety is inversely related to speed. The road speed limits are established by Traffic Nation Rules. The current criteria for establishing the speed limit is controlled by a wide range of aspects including

- Driver aspects (e.g., handling skill, body response time, attention, concentration, vision acuity, forward visibility, light reflection, danger perception, comfort, fatigue, age, and trip expenditure);
- Vehicle properties (e.g., handling, suspension response, brake performance, window visibility, tire grip, maintenance conditions, and cost);
- Road aspects (e.g., road class, road quality, line number, track segregation, asphalt adhesion, surface condition and contamination, roughness, stagnant water, roadway geometry, sinuosity, signaling structure, horizontal visibility, maintenance conditions, sight distance, crossings, pedestrians, cyclists, and obstructions);
- Weather condition (e.g., lighting, visibility, reflection, clouded, temperature, wind, mist, rain, snow, ice, and leaves);
- Environmental affect (e.g., traffic density, traffic mixture, other moving bodies closeness, fuel consumption, pollution, noise, and road neighbors); and
- Other aspects (e.g., social, cultural, historical, political, heuristic, actual experimental observed speed, road tax, insurance value coverages and bonuses, driver risk acceptance, guesses, and intuitive judgments).

Some of the aspects mentioned in this list can be related to a specific social segment. Vehicle performance is associated with the competence of the automotive industry, the road quality is a public administration responsibility, and driver handling skill is controlled by the federal authority for driver's licenses. Others aspects have a common sense merit and are quantitatively imponderable or uncontrollable (e.g., the weather). Therefore, whatever the methodology used to study this subject, the result will likely produce something diffuse and prolix, poorly reliable nor creditable, rather than a universal, believed rational, scientifically-based recommendation.

According to Hauer,

...the evolution of speed over time is poorly documented, and the understanding of what drives the evolution is largely missing. It is known that speeds evolve over time, but not why. Although the prevalent and strongly held belief that the greater the speed, the higher is the probability that accidents will occur is, at present, not well supported by research... (Hauer 2009). According to Mannering,

...it has become more common for speed limits to be set for political reasons rather than for safety reasons. As a consequence, the motoring public seems to have increasingly begun questioning the rationality or speed limits. This is evident in observed speed data that show that the majority of drivers routinely exceed posted speed limits. A key motivating factor in drivers' tendency to exceed the speed limit is that they believe that the excess speed does not threaten safety... (Mannering 2009). It can be observed from different writers' opinions that a credible deterministic technical rule, with scientifically proven reliability, is necessary to stake public authority to deal with the subject. Also, according to Elvik, in Norway and Sweden, an optimal speed limit is set to minimize the total costs of transport to society. Travel time, vehicle operating costs, road accidents, traffic noise, and air pollution were considered in the determination of optimal speed limits (Elvik 2002).

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A vehicle's capability to travel properly on different road pavement conditions is a desirable characteristic that is tuned by vehicle design engineering. Robust active suspensions (Wang et al. 2008) and global motion control technology (Costa 1992) are examples of recent developments. Actually, the traffic engineer is able to quantitatively calculate speed limits only for horizontal curves (Baerwald 1976). This is established from the car's centrifugal inertia force balanced with the transversal tire/road force, which is dependent on the curve radius, superelevation, and friction coefficient. However, it is difficult to establish proper values for straight line driving. The driver expects freedom for his own driving style, and prudence recommends safety in vehicle speed.

The motivation of this work is to contribute scientifically to the complex task of identifying vehicle speed limits, concerning the safety aspects of modern highway systems. The proposed methodology to study this subject is the evaluation of vehicle vertical behavior and relating it to superficial pavement irregularities (Barbosa 1998; Silva et al. 1999; Sun et al. 2001; Sun and Luo 2007), based on a dynamic vehicle/road interaction model. Vehicle safety is the ability to handle adequately the vehicle on the road. Handling quality is a vehicle property that allows a driver to accelerate and stop, to curve and corner, in short, to manage vehicle trajectory through fast and heavily trafficked modern highways. Vehicle driveability is directly related to a vehicle's capability of producing tire horizontal forces. For acceleration and deceleration purposes, longitudinal tire force is required. For lateral maneuvers and curve cornering, lateral tire force is needed. These forces however, are limited to the Coulomb's relationship that correlates vertical tire/road contact forces to tangential forces. This relation is expressed by the tire/road friction coefficient. Therefore, what is desirable is the minimum variation of contact forces that allows the largest horizontal force available, for a given friction coefficient.

To this end, a quarter-car vehicle model was derived, and the vertical elevation of an unpaved road surface was measured. As the road profile roughness is unique to each section, a statistic description was used to qualify this random process. Both functions on the frequency domain were used to statistically define the proneness of vehicle handling capability in a function of the traveling speed.

Vehicle Modeling

The model used in this research is the quarter-car vehicle model (Barbosa 1998). The body and suspension's vertical movements are described by the x_1 and x_2 coordinates. The two degrees of freedom lumped vertical model is shown in Fig. 1. Mass m_2 represents the sprung vehicle body with secondary suspension (k_2 and c_2). Mass m_1 corresponds to the primary suspension composed of tire stiffness and damping (k_1 and c_1).

This model is base-excited by pavement irregularities that induce suspension movements. The vehicle motion equations are obtained by applying Newton's second law to masses m_1 and m_2 , resulting in the following differential equations:

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{u}) + k_1(x_1 - u) - c_2(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) = F_1 \quad (1)$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = F_2 \quad (2)$$

Various vehicle frequency response functions may be obtained from this traditional model. For the purpose of this research, the vertical contact force is the objective function to be evaluated.

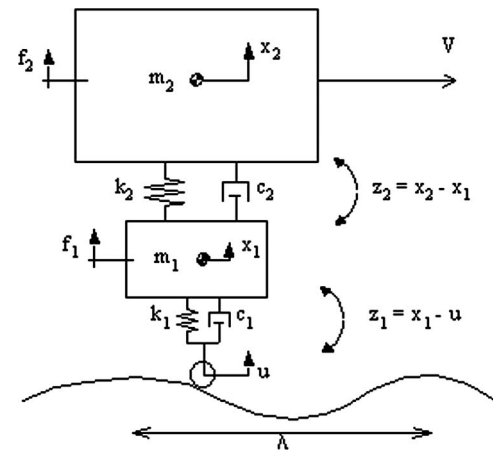


Fig. 1. Quarter-car vehicle model over pavement evenness

The vertical wheel contact force, f_c , between tire and pavement is obtained from the primary suspension deflection and velocity. Considering no external forces acting on the car body and choosing a special pair of coordinates (i.e., $z_1 = x_1 - u$ and $z_2 = x_2 - x_1$), the following can be obtained:

$$m_1(\ddot{z}_1 + \ddot{u}) + c_1\dot{z}_1 + k_1z_1 - c_2\dot{z}_2 - k_2z_2 = 0 \quad (3)$$

$$m_2(\ddot{z}_2 + \ddot{z}_1 + \ddot{u}) + c_2\dot{z}_2 + k_2z_2 = 0 \quad (4)$$

$$c_1\dot{z}_1 + k_1z_1 = f_c \quad (5)$$

By taking the Laplace transforms of these differential equations and assuming zero initial conditions, one obtains

$$(m_1s^2 + c_1s + k_1)Z_1(s) - (c_2s + k_2)Z_2(s) = (-m_1s^2)U(s) \quad (6)$$

$$(m_2s^2 + c_2s + k_2)Z_2(s) + (m_2s^2)Z_1(s) = (-m_2s^2)U(s) \quad (7)$$

$$F_c(s) - (c_1s + k_1)Z_1(s) = 0 \quad (8)$$

Eliminating z_1 and z_2 from these equations and after some algebraic manipulation to sort the wheel contact force $F_c(s)$ per displacement excitation $U(s)$ relationship, the vehicle deterministic normalized wheel contact force transfer function, $H(s)$, is obtained

$$\frac{F_c(s)}{U(s)} = \frac{(c_1s + k_1)[(m_2s^2 + c_2s + k_2)(m_1s^2) + (m_2s^2)(c_2s + k_2)]}{[(m_2s^2 + c_2s + k_2)(m_1s^2 + c_1s + k_1) + (m_2s^2)(c_2s + k_2)]} = H(s) \quad (9)$$

The function $H(s)$ has a fifth-order polynomial in the numerator (s^5) and a fourth-order denominator (s^4). Analyzing the frequency domain response, replacing s with $i\omega$, and assuming the vehicle properties defined in Table 1, the modal properties are obtained

Table 1. Inertial and Suspension Quarter-Car Characteristics

Characteristics	Vehicle body	Suspension	Proportion (usual)
Mass	375.0 kg	30.0 kg	12% (10 ~ 15%)
Rigidity	18.25 kN/m	146.00 kN/m	$8 \times (5 \sim 10 \times)$
Damping	1,825 Ns/m	182.50 Ns/m	10%(5 ~ 10%)

Note: Total vehicle normal load N : 4,050 N.

(eigenvalue and eigenvector), and the frequency response curve, $H(\omega)$, normalized by the vehicle normal load, N , can be drawn, as shown in Fig. 2.

The quarter-car model presented in Fig. 1 with two bodies (the vehicle body and the hub/wheel) has a modal behavior with two vibration modes. The first vibration mode occurs when the two bodies move in the same direction (i.e., the x_1 and x_2 coordinates vary in phase). This vibration mode is associated primarily with large vehicle body movement (a magnitude of the second degree of freedom presented in Table 2 for the first mode). The second vibration mode occurs when the two bodies move in opposite directions (i.e., the system coordinates vary out of phase), which is associated with suspension movements (a large movement of the unsprung mass of the hub and wheel presented in Table 2 for the second mode) (Barbosa 1999; Barbosa and Costa 2001).

The vehicle frequency response function, $H(\omega)$, has two natural damped modes with frequencies 1.1 and 12 Hz, with damping factors of 0.30 and 0.47 for first and second modes, respectively. The normalized eigenvectors obtained from the system matrix, describing the modes proportion and phase angle, are presented in Table 2.

For example, analyzing the wheel contact force transfer function, $H(s)$, presented in Fig. 2 and focusing only into the vehicle's first mode of vibration around 1.1 Hz, a vehicle with a speed of 10 m/s, traveling on a periodic surface with a 10 m wavelength and a 0.01 m amplitude, reveals that the tire contact force will vary $\pm 9\%$ (magnitude of the transfer function $M = 9.5$) around the nominal vertical load. This affects the vehicle's handling properties, reducing its ability to generate guidance tangential forces.

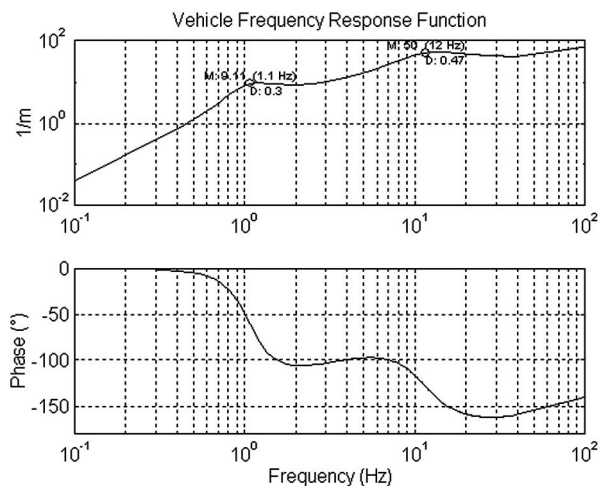


Fig. 2. Vehicle normalized contact force transfer function

Table 2. Modal Eigenvalue and Eigenvector

Vibration mode number	Mode 1 (vehicle)		Mode 2 (wheel/hub)	
Damped natural frequency	1.1 Hz		12.0 Hz	
Damping factor	0.30		0.47	
Degree of freedom	Magnitude	Phase	Magnitude	Phase
x_1	0.1471	-34°	0.9999	0.0°
x_2	0.9999	0.0°	0.0138	-235°

Pavement Roughness Measurements

Road roughness is used as vehicle excitation. An unpaved, 1,400 m long road section of rustic soil pavement with some spread gravel was measured. The surface elevation was measured with a mobile three-point-middle-chord system (Pavimetro Measuring System). This system is composed of a structure and three wheels. The two external wheels are steerable and the central one is movable in relationship to the others. A conventional car pulls the system along the road to measure the track elevation. The central wheel vertical movement is monitored with a precision displacement transducer (LVDT), and elevation is sampled every centimeter. An analog to digital sample board installed in a portable computer acquired the data that are stored in magnetic media for post processing. The vertical unpaved road profile measured is presented in Fig. 3.

The measured roughness data are treated with the measuring system transfer function to obtain the topographic vertical elevation (Pavimetro 2009), as presented in Fig. 3. In this case, 10 points per meter were sampled (i.e., one sample every 0.1 m). A bad roughness shape is shown.

Because road evenness is not a deterministic function (each road section has a different shape) a statistical random process representation was used. The road irregularity described in the space domain was transformed to the frequency domain. The pavement elevation measurements were manipulated to obtain the distribution for the wavelength of periodic irregularities (i.e., an evenness signature). This treatment was performed for 2,048 points at 10^{-2} m intervals. This range allows analyzing the wavelength for 200 m at 0.1 m intervals. The power spectral density function (PSD) between 0.1 and 10 m wavelength of the soil vertical elevation is presented in Fig. 4.

In analyzing the result of the irregularity spectrum distribution, a particular signature was observed with an intensified density magnitude in the wavelength range of 0.5–0.9 m. The spectral density of the measured pavement roughness was equivalent to the ISO quality Level C (ISO 1995) for wavelengths longer than 5 m [2.56×10^{-4} at 10 m wavelength, as shown in Fig. 4].

As discussed by Andr n (2006), the spectral description of surface roughness has a different slope proposition. Despite the ISO standard to qualify pavement roughness with a single slope, in fact, what is really observed in measured pavement roughness

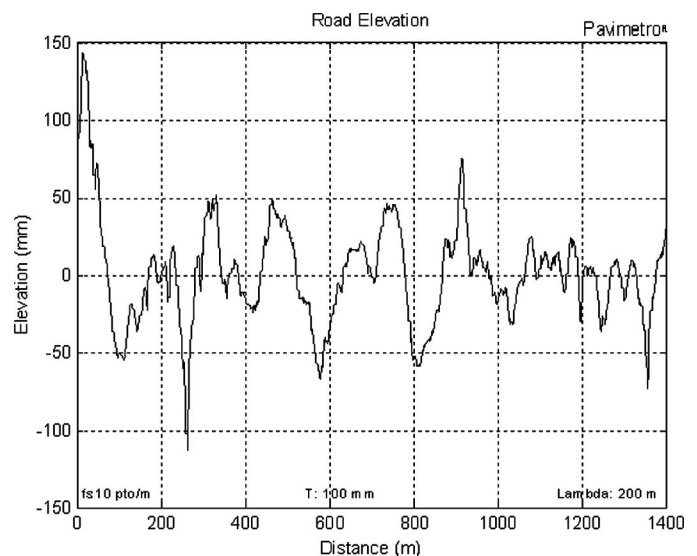


Fig. 3. Surface elevation measurement of the unpaved road

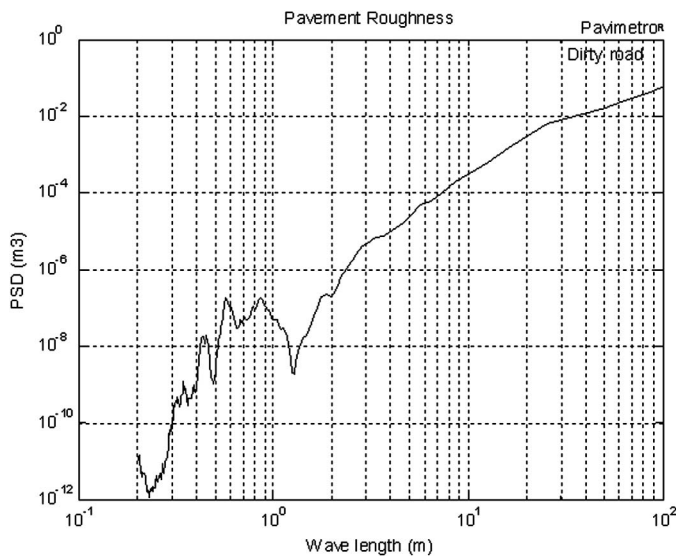


Fig. 4. Power spectral density roughness of the unpaved road

is a content wave number of a density less than 2 dB (decibel) per decade above 0.5×10^1 1/m (or shorter than wavelength 2.0×10^{-1} m) and seems to have two or more slope variations.

Speed Limit Concept

One contribution to establish the safety speed limit can be based on the concept of the loss of vehicle driving abilities. Handling a vehicle is the ability to produce demanded horizontal contact force to control vehicle acceleration and attitude. The maximum tire horizontal contact force is defined by Coulomb's relationship, $F \leq \mu N$. For a small vertical wheel/road contact force, the available tangential guidance force is proportionally affected. The extreme situation is when the wheel/road vertical contact force is null (loss of contact), preventing the possibility of generating tire tangential force. Therefore, the higher and less variable the vertical tire load is, the larger the capacity to generate tangential guidance tire force, ensuring safety.

Vehicle/Road Relationship

To evaluate the vehicle vertical time response attributable to excitation by pavement irregularities, the convolution of the vehicle function with the surface vertical displacement profile can be used (Newland 1984). However, in the frequency domain, the spectral response may be obtained directly only by multiplying the spectrum of the two systems. The resulting function is the inertance magnification of vehicle contact force attributable to spectral pavement roughness. Therefore, considering the vehicle as a time invariant linear system and the pavement roughness as a random input with Gaussian probability distribution, the block diagram presented in Fig. 5 shows function relations in the frequency domain.

Natural vehicle behavior can be expressed as its frequency (1/time) domain function. Pavement irregularities are expressed

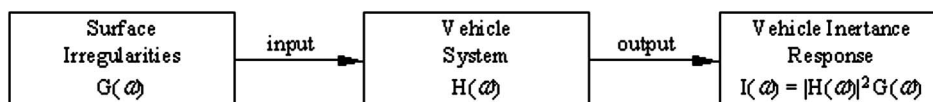


Fig. 5. Flow block diagram

in spatial frequency (1/space). The relationship between time frequency, ω , and spatial frequency, n , is the vehicle speed, V , simply expressed by

$$\omega = V \cdot n \quad (10)$$

where ω = frequency in hertz; n = spatial frequency in 1/m; and V = vehicle speed in m/s.

Considering pavement irregularity as an ergodic stationary random process (i.e., it has statistical properties obtained from a single, sufficiently long sample) with normal distribution, the evenness density can be expressed through the root mean square (RMS), in units of roughness. According to Parseval's theorem (Oppenheim and Schaffer 1975), the RMS density of a normally distributed random vertical displacement roughness is the square root of the power spectral density. Transforming $G(n)$ into the frequency domain with the expression 10, the pavement roughness frequency function, $G(\omega)$, is obtained as

$$G(\omega) = V \cdot G(n) \quad (11)$$

Finally, in the frequency domain, the vehicle vertical contact force density function, $I(\omega)$, is obtained from the squared vehicle inertance function weighed by the pavement roughness spectrum (Felício 2007)

$$I(\omega) = |H(\omega)|^2 G(\omega) \quad (12)$$

The results for the inertance function at 25 km/h are presented in Fig. 6.

The pavement roughness spectral density function has been extended at a rate of 5 dB/decade after the maximum acquisition wave number just for length compliance purposes.

The tire/pavement contact force function reveals system characteristics. Considering pavement irregularities as a broadband signal, all vehicle frequencies may be excited at different speeds. Restricting ourselves only to the human perception frequencies range (between 0.1 and 100 Hz), the amplification contact load may be obtained at a given speed. Considering that the input has a Gaussian

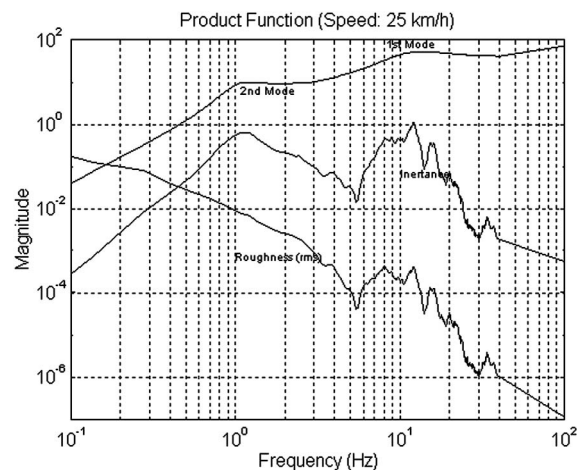


Fig. 6. Inertance function

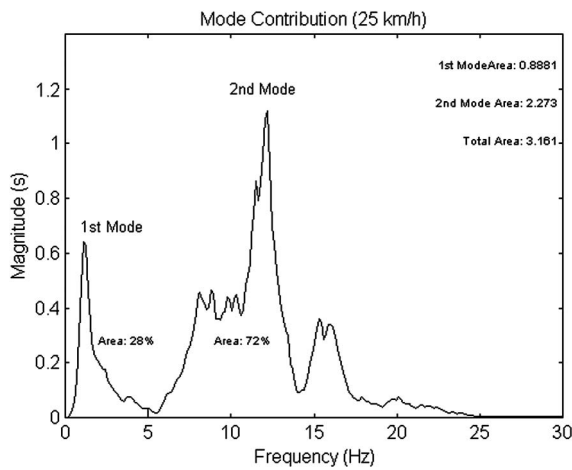


Fig. 7. Mode contribution of the inertance function at 25 km/h

probability distribution (Newland 1984), the density function, $I(\omega)$, can be integrated over all the selected frequency ranges for a given speed, resulting in the contact force probability distribution, F_c , expressed by

$$F_c = \left[\int_{\omega_{\min}}^{\omega_{\max}} I(\omega) \right]^{0.5} \quad (13)$$

As an example, Fig. 7 depicts the modal contribution to the product function, $I(\omega)$, for a vehicle traveling at 25 km/h (6.7 m/s). The first mode contribution represented by the area below the curve reaches 28%, whereas the second mode contribution is almost two and a half times larger (72% of the area below the curve). This fact is assigned to the synchronism of the pavement evenness wavelength (between 0.5 and 0.9 m) at this speed and the second mode natural frequency (approximately 12 hertz). The mode separation is settled in this case to 5 Hz.

In evaluating the function $I(\omega)$ for each speed up to 50 km/h (13.8 m/s), the overall magnitude of the function (i.e., the sum) and the contribution of each mode are determined. The probability of the normalized contact force variation [i.e., the area below the $I(\omega)$ function] as a function of the vehicle speed is presented in Fig. 8.

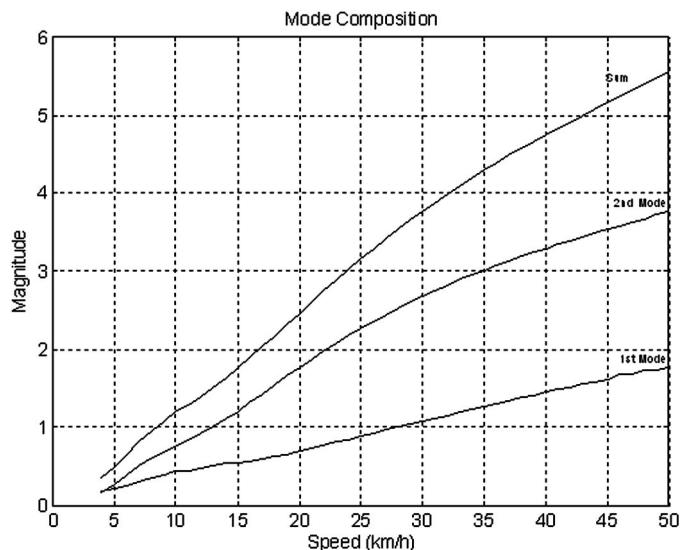


Fig. 8. Vehicle speed and contact force

Considering the measured pavement roughness as a Gaussian process, the likelihood of a wheel to take off with 99.73% confidence (three times the magnitude in the standard deviation scale) is when the vehicle speed is faster than 25 km/h, as shown in Fig. 8. Considering that the loss of vertical contact force is not acceptable (i.e., the magnitude must be less than three) to guarantee the possibility of generating lateral forces (i.e., driveability), and the maximum safe speed limit for this vehicle traveling over this pavement condition is quantified.

Conclusion

In this research, dynamic vehicle behavior and pavement roughness were used to define safety traffic speed limits. The methodology adopted was based on the vehicle vertical wheel contact force frequency response excited by measured pavement irregularities. A quarter-car model was used to identify vehicle dynamic behavior. The vertical elevation of an unpaved road surface was measured. The roughness spectral density was quantified as ISO Level C. Calculations for the vehicle inertance function in the frequency domain with the vertical contact force transfer function were derived with the pavement spectral density roughness function. The statistical contact load variation was obtained from the vehicle inertance density function integration over a frequency range. The vehicle safety behavior concept was based on the ability to generate tangential forces in the wheel/road contact interface. This handling ability was proportional to tire/pavement vertical contact forces. The traffic safety speed limit was obtained from the likelihood of the loss of vehicle driveability. The results showed that faster than 25 km/h the likelihood of contact loss is 99.7% on the measured unpaved road.

Because of vehicle vibration modes, a typical pavement wavelength is harmful for a given traffic speed. This wavelength also makes the worst tire/road load magnification that increases damage to the pavement structure. These irregular wavelength ranges should be minimized or eliminated during pavement maintenance.

The model used restrictions in the analysis of a linear case. A real vehicle suspension has displacement limitations (i.e., it has end stop rubber bushing). This nonlinear characteristic can be incorporated only by using a time integration solution. Additionally, tire diameter, tread size, and internal pressure can soften the contact area, modifying the tire contact length value (typical 0.10 m) and short-wavelength filtering.

A simple two degrees of freedom vertical vehicle model was used for this study. However, on the basis of this methodology, a more detailed model can be employed to access the whole vehicle body movements. Particularly, for vehicle cornering behavior, it would be of interest to access curving speed limits, considering pavement superelevation and irregularities. More than the speed limit indication, this scientific approach may also be useful for the evaluation of passenger comfort, the determination of a vehicle's suspension and structural design, and as a vehicle/road interaction tool to evaluate harmful pavement roughness.

Road irregularities are considered as a single-track patch (i.e., only one tire contact point). Multiple axle/wheel models may be analyzed considering a time delay from the front input of $\Delta t = l/V$ in relation to rear inputs, where l = wheelbase. A complete four wheel independent suspension vehicle model may be employed to access the dynamic behavior provided by a correlation between the irregularities of the left and the right track sides.

Finally, the scientific contribution of this work is the proposition of an objective and quantitative technique to assist the establishment of speed limit on the basis of vehicle/road interaction.

The vehicle dynamic response and the road evenness are analyzed together on the frequency domain, contributing to the identification of safe conditions.

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