

The influence of suspension components friction on race car vertical dynamics

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ABSTRACT

This work analyses the effect of friction in suspension components on a race car vertical dynamics. It is a matter of fact that race cars aim at maximising their performance, focusing the attention mostly on aerodynamics and suspension tuning: suspension vertical and rolling stiffness and damping are parameters to be taken into account for an optimal setup. Furthermore, friction in suspension components must not be ignored. After a test session carried out with a F4 on a Four Poster rig, friction was detected on the front suspension. The real data gathered allow the validation of an analytical model with friction, confirming that its influence is relevant for low frequency values closed to the car pitch natural frequency. Finally, some setup proposals are presented to describe what should be done on actual race cars in order to correct vehicle behaviour when friction occurs.

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Nomenclature

| | |
|----------|---|
| NVH | noise, vibration and harshness |
| 2 DoF | two degree of freedom |
| 4 DoF | four degree of freedom |
| F_{st} | Stribeck friction force (N) |
| F_s | maximum static friction force (N) |
| v_s | Stribeck sliding speed coefficient |
| k_v | Stribeck viscous friction coefficient (Ns/m) |
| i | Stribeck exponent |
| M | sprung mass (kg) |
| m_F | front unsprung mass (kg) |
| m_R | rear unsprung mass (kg) |
| I_{YY} | sprung mass pitch inertia (kgm ²) |
| k_F | front suspension spring stiffness (N/m) |
| k_R | rear suspension spring stiffness (N/m) |
| k_{tF} | front tyre stiffness coefficient (N/m) |
| k_{tR} | rear tyre stiffness coefficient (N/m) |

| | |
|----------|--|
| c_F | front non-linear suspension damping coefficient (Ns/m) |
| c_R | rear non-linear suspension damping coefficient (Ns/m) |
| c_{tF} | front tyre damping coefficient (Ns/m) |
| c_{tR} | rear tyre damping coefficient (Ns/m) |
| F_{dF} | model front friction force (N) |
| F_{dR} | model rear friction force (N) |
| x_M | sprung mass vertical displacement (m) |
| θ | sprung mass pitch angle (deg) |
| x_{MF} | front sprung mass vertical displacement (m) |
| x_{MR} | rear sprung mass vertical displacement (m) |
| x_{mF} | front unsprung mass vertical displacement (m) |
| x_{mR} | rear unsprung mass vertical displacement (m) |
| v | relative velocity (m/s) |
| $y(t)_F$ | front input vertical motion (m) |
| $y(t)_R$ | rear input vertical motion (m) |

1. Introduction

It is widely acknowledged that nowadays vehicle modelling is an extremely useful tool for a wide range of automotive applications. For example, models are developed to design and integrate several road vehicles subsystems, and to estimate the overall vehicle NVH (noise, vibration and harshness characteristics), efficiency and fuel consumption. While road vehicles engineers focus their attention mostly on vehicle active safety, race engineers aim at maximising vehicle performance in terms of longitudinal/lateral dynamics [1] and lap time. About that, in recent times virtual models, simulations and indoor testing have become an essential factor for motorsport application as well. Since several championship rules reduce the track test sessions during the entire season and rigidly limits the amount of time a team can spend on indoor testing facilities (i.e. Four Poster), all previous analysis proves to be fundamental.

As a first step, simple analytical models represent a relevant tool for an easy and fast understanding of vehicle response. The Two DoF Quarter Car model is commonly used for preliminary ride analysis [2], for the study of random vibration response [3,4] and to verify new components/approaches benefits on vehicle behaviour [5,6]. Despite that, the Four DoF Half Car model represents a further evolution in vehicle analytical models, introducing the sprung mass pitch motion [7] and both front and rear unsprung masses degrees of freedom. In case of race cars, unsprung masses have a huge influence in vertical dynamics as tyre stiffness is very close to suspension stiffness (the race car tested in this work features a 140 N/mm spring vertical stiffness with a 130 N/mm tyre vertical stiffness in running conditions). The pitch angle must be considered in ride analysis as well: together with bounce motion it deeply affects vertical dynamics behaviour [8], an uncontrolled pitch response on the other hand modifies a race car aerodynamic balance.

In order reproduce actual car response, all the models discussed above need to take into account as much details as possible. Tyre characteristics and thermal influence, components non-linearity and friction are examples of how these models can be made more specific. Dealing with friction, it is well known that whichever solution is adopted the

relative motion among the suspension components is unavoidably affected by friction phenomena. Typically, it derives from the suspension arms joints and bushes either on the chassis side or on the wheel side, and from the shock absorbers internals. A literary survey on this topic has shown that it is very rare to find works that consider friction phenomena and its effects in race car applications. In fact, such vehicles feature lighter masses than typical road vehicles and a suspension stiffness vs. tyre vertical stiffness ratio close to 1. Hence, friction may act in a different frequency range directly affecting race car performances.

Lots of friction models have been proposed in order to predict and evaluate the friction influence on the performance of generic mechanical systems. Amongst them, the basic Coulomb–Viscous model represents friction as a constant force value [9], while the Stribeck model defines friction as the sum of higher static friction and dynamic sliding friction [9]. On the other side, the LuGre model [10,11] and its evolutions take into account friction dynamic effects expressing them with first-order differential equations.

As regards transportations, Wu [12] presented a review of friction wedge suspension for rail vehicle with three-piece bogies where friction is used as a damping factor. In the automotive field, instead, Lizarraga [13] proved that the Stribeck effect is present in the sole suspension shock absorber element. Considering the entire vehicle, studies on road vehicle riding comfort were carried out. Mikhailov proposed a new friction model [14] as a combination of elastic and viscous friction for the study of the vibro-damping characteristics of a vehicle seat. It is interesting to notice that the focus of all these studies lies on road vehicles and passengers comfort. No one has analysed friction effect on a race car vertical and handling dynamics yet.

The introduction of friction in race car simulation can contribute to a reduction of the error between virtual model and actual vehicle response, giving engineers the possibility to better predict the real behaviour and properly tune the suspension system when friction occurs.

2. Objective

In this work a race car was tested and analysed on a Four Poster rig [15]. The analysis showed a high level of friction, especially in the front suspension system. Dealing with race cars, friction is a relevant factor in terms of performance and it is clearly visible because of the overall light weight of sprung and unsprung masses when compared to road vehicles. Although the friction phenomenon decreases with vehicle mileage, in several feeder formula series lots of light and heavy crashes happen because of drivers' inexperience. When this happens, a new suspension set must be installed causing friction to go back to its initial amount.

Thanks to virtual model validation by means of real data, a comparison of suspension damping tuning with and without friction is shown in order to highlight the unavoidable influence of friction. Hence, the final aim of this work is to evaluate the level of friction influence in the vehicle vertical dynamics behaviour, showing when friction is relevant in vehicle vertical response. Finally, some setup proposals are presented in order to describe what should be done to correct vehicle behaviour when friction occurs.

3. Methodology

In order to verify the actual influence of friction in vehicle vertical response and to estimate how the overall suspension damping is affected by that, the methodology below has been followed:

- (1) Vehicle testing on Four Poster (Figure 1): the vehicle was tested with a bounce sweep vertical displacement input in order to perform a frequency domain analysis and to evaluate vehicle response for either bounce and pitch motions. Furthermore, a step vertical displacement input was applied to the four actuators simultaneously with the aim to estimate the transient response.
- (2) Analytical vehicle model: several friction models for suspension system were analysed and embedded in a Half Car vehicle model. The simulation allowed the identification of a detailed friction model and a first estimation on how friction affects vehicle behaviour.
- (3) Model validation results: real world data were compared to a virtual model with and without friction results. Quantities as vertical and pitch accelerations, tyre load variations and spring compressions were monitored either in frequency domain or time domain.
- (4) Result analysis: starting from the previous comparison, friction influence was then analysed and adjustment for situations in which friction occurs were finally proposed.

3.1. Vehicle testing on Four Poster

The Four Poster rig is an indoor test facility widely used in automotive field for vehicle ride characterisation [15]. Before the actual dynamic tests, the reference vehicle was equipped with a data acquisition system and sensors capable of gathering information



Figure 1. Real vehicle on the Four Poster rig.

regarding sprung and unsprung masses vertical accelerations, front and rear suspension displacements and tyre vertical load variations. Then a reference static setup was defined and the car was weighed, in order to exactly assess the main parameters values and their behaviour and to set the virtual model correctly.

The car was placed on the Four Poster bench for the dynamic procedure: it was subjected to a preliminary warm-up phase consisting of a 4 mm displacement sine input motion at a 4.5 Hz frequency until the front and rear dampers reached a working temperature of 50°. For the ride analysis, two different laws of motion were applied: a 0–30 mm displacement step input for a time domain analysis and a constant velocity amplitude sine sweep displacement law of motion, in a frequency range of 0–25 Hz, for a frequency domain analysis. As stated before, the suspension system showed an unavoidable level of friction, mostly due to suspension arm joints to the chassis pick-up points, to the steering rack, to the rockers and dampers internals. As a standard, a set of high performance rod ends is fitted for each suspension and steering components in order to avoid any undesirable compliance. However, the absence of clearance between the single rod end ball and its case implies a high friction resistance along the suspension motion, and the total contribution of all joints clearly affects the global suspension motion at least until the rod ends reach a certain mileage.

3.2. Analytical vehicle model

3.2.1. Friction models investigation

Friction is known as a resistance to relative motion when two surfaces slide against each other. Even if it can be considered useful for many applications – like driving, cornering and stopping a vehicle – it can lead to undesirable effects. For this reason, several friction models have been developed and described in literature in order to evaluate the influence of friction on the dynamic behaviour of generic mechanical systems. On those basis, four different models have been embedded into the following virtual simulations.

The Coulomb is the most basic friction model. It is used to describe friction either in dry contacts and mixed lubricated contacts as well. As the change in direction of velocity can lead to instability, this model is often troublesome from the simulation point of view, and a combination with the Viscous model is presented in literature as an improvement. Either the Coulomb and the Coulomb–Viscous models present a constant value of friction force after the relative velocity change in sign. However, even dry contacts may behave similarly to lubricated contacts, where the friction decreases with increased sliding speed and the value of static friction is higher than dynamic friction. This effect is accurately described by the Stribeck friction model [9,12], which calculates the friction resistance force as in Equation (1):

$$F_{st} = (F_c + (F_s - F_c)e^{-(|v|/v_s)^i})\tanh(k_{\tanh}v) + k_v v, \quad (1)$$

where F_{st} is the resulting Stribeck friction force, F_c is the above mentioned Coulomb sliding force, F_s the maximum static friction force, v_s the sliding speed coefficient, k_{\tanh} is the velocity coefficient, k_v the viscous friction coefficient. The introduction of the tanh function prevents instability problems when the sliding velocity reverses its sign. At a high value of this coefficient it is possible to reproduce the effect of different static and dynamic friction for very low velocities, avoiding the computation problems derived from the change in sign.

As discussed above, the transition from positive to negative velocities could lead to numerical instability for some of the above friction models. This change in direction is always associated with a small relative tangential displacement between interacting surfaces. Hence, it is worth to consider a friction model that can describe this effect accurately.

Usually, the models that better represent micro-displacements are formulated as a differential equation. One of the most popular micro-slip friction models is the so-called LuGre model, which includes many characteristics of the friction models previously described. LuGre describes the friction phenomenon from a microscopic point of view, assuming that it is generated by the contact of the surface roughness. The micro asperities can be modelled as bristles, which behave like a spring when there is a relative velocity [11].

With regard to vehicle suspension systems, Mikhailov [14] proposed a new model to study the vibro-damping characteristics of suspended vehicle seats and the related friction characteristics.

All these models have been used in virtual simulations but only the Stribeck model is detailed above as the simulations performed shown that this is the best approximation of friction phenomenon in suspension systems.

3.2.2. Four DoF Half Car model with friction

As stated above, the two DoF Quarter Car model is a simple but effective tool for preliminary ride analysis even if it represents only a single corner of a vehicle and it is not as reliable as other models for a more detailed vertical dynamics evaluation. Consequently, it has been replaced by the Half Car model (Figure 2), that introduces further degrees of freedom. In particular, considering a single sprung mass and both the front and rear unsprung masses, it is possible to investigate either the sprung mass bounce response and the pitch response, relevant characteristics in terms of comfort vs. handling compromise.

Thus, the Equations (2)–(5) describe the motion of the Sprung and Unsprung masses:

$$I_{yy}\ddot{\theta}_M - a \cdot c_F(\dot{x}_{MF} - \dot{x}_{mF}) - a \cdot k_F(x_{MF} - x_{mF}) - a \cdot F_{dF} + b \cdot c_R(\dot{x}_{MR} - \dot{x}_{mR}) + b \cdot k_R(x_{MR} - x_{mR}) + b \cdot F_{dR} = 0, \quad (2)$$

$$M\ddot{x}_M + c_F(\dot{x}_{MF} - \dot{x}_{mF}) + c_R(\dot{x}_{MR} - \dot{x}_{mR}) + k_F(x_{MF} - x_{mF}) + k_R(x_{MR} - x_{mR}) + F_{dF} + F_{dR} = 0, \quad (3)$$

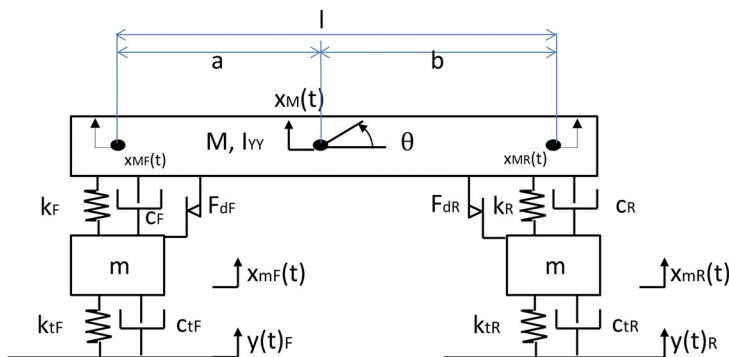


Figure 2. Half Car model.

$$\begin{aligned}
m_F \ddot{x}_{mF} - c_F (\dot{x}_{MF} - \dot{x}_{mF}) - k_F (x_{MF} - x_{mF}) - F_{dF} + c_{tF} (\dot{x}_{mF} - \dot{y}(t)_F) \\
+ k_{tF} (x_{mF} - y(t)_F) - F_{dF} = 0,
\end{aligned} \tag{4}$$

$$\begin{aligned}
m_R \ddot{x}_{mR} - c_R (\dot{x}_{MR} - \dot{x}_{mR}) - k_R (x_{MR} - x_{mR}) - F_{dR} + c_{tR} (\dot{x}_{mR} - \dot{y}(t)_R) \\
+ k_{tR} (x_{mR} - y(t)_R) - F_{dR} = 0,
\end{aligned} \tag{5}$$

where M and I_{yy} denote the sprung mass and Inertia, m_F and m_R are the front and rear unsprung masses respectively, a and b represent the front and rear axis longitudinal distance from GC location, k and k_t denote the springs and tyre stiffness (for both front and rear axis), while c and c_t are the shock absorbers and tyres damping coefficients (for both front and rear axis), the latter often considered to be negligible. θ angle is the pitch degree of freedom, strictly related to the sprung mass displacements x_{MF} and x_{MR} and to the CG displacement x_M as reported in Equations (6) and (7):

$$x_{MF} = x_M - a \cdot \theta, \tag{6}$$

$$x_{MR} = x_M + b \cdot \theta. \tag{7}$$

The F_d terms refer to the friction effect embedded into this model for both front and rear axis. Starting from the above detailed equations, a Half Car model has been developed by means of MATLAB Simulink package.

In order to model the entire system as faithfully as possible, the shock absorber damping action was represented as a non-linear function of the sprung and unsprung mass relative velocity for both front and rear suspensions. It was thus possible to detail the typical damper force vs. speed characteristic, either for bump (compression) and rebound (extension) motions, starting from the manufacturer data obtained for several dampers configurations and reported in Figure 3. Furthermore, even the spring stiffness values were modified into a non-linear curve in relation to the different front and rear installation ratio curves, obtained after a detailed suspension kinematic analysis. Tyre stiffness and damping were detailed as well: the manufacturer data in fact report these characteristics as function of tyre inflating pressure and camber angle, as shown in Figure 4.

Thanks to the versatile Simulink block programming structure, it was possible to build the previously described four friction models separately, embedding them into both the front and rear equations and simulating them, one at a time, with different configurations. The choice of each friction model parameters was carried out with reference to the above mentioned authors and adjusted through an iterative ‘trial and error’ approach, based on the root mean square error between virtual model results and real data. Subsequently, the same road profiles inputs $y(t)$ used on the Four Poster rig were then applied to the Half Car model in order to compare model response with real-life data. The calculation of dynamic wheel load variation was then added to the model for a complete comparison of all the data gathered. As in most cases [17,18], even in this system the bounce and pitch motions are not pure but coupled: the coupled bounce frequency is around 3 Hz and occurs as a front end bounce motion, while the coupled pitch frequency results around 6 Hz and occurs as a rear end bounce.

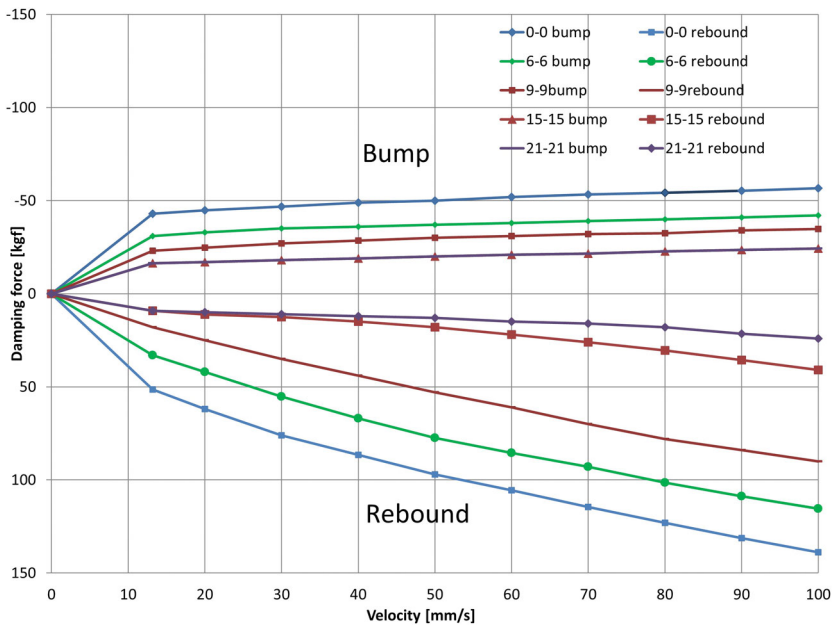


Figure 3. Non-linear damper curves given by the manufacturer (as reported in [16]).

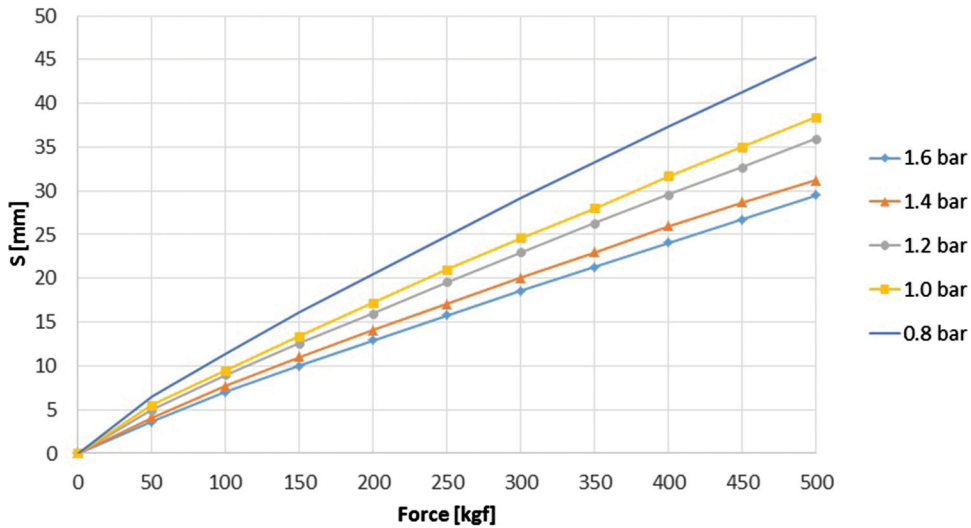


Figure 4. Pirelli tyre front vertical stiffness for -3° negative camber angle as function of several inflating pressure (as reported in [16]).

3.3. Model validation results

Figures 5–8 show the Half Car model validation through the experimental data collected. The introduction of friction has a corrective effect on the analytical model, showing that this phenomenon plays a key role in a suspension system. Friction improves the capability of virtual simulations to represent the actual data in the whole input frequency range. The Stribeck friction model turned out to be the best description of suspension friction.

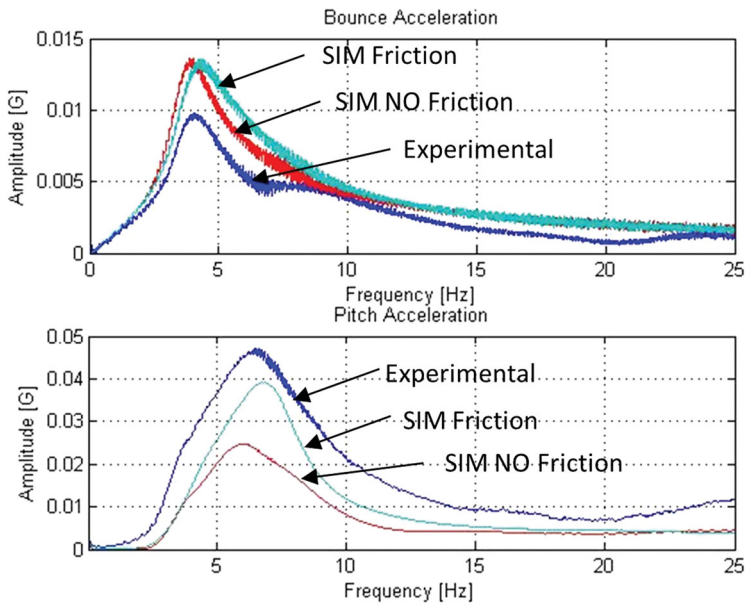


Figure 5. Bounce and pitch acceleration FFT for the Half Car model.

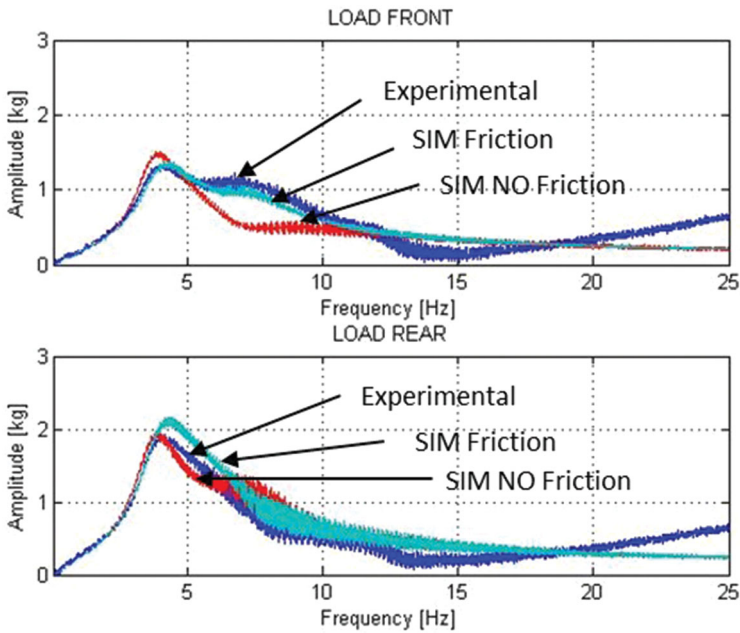


Figure 6. Front and rear tyre load variation FFT for the Half Car model.

From the accelerations and tyre load variation fast Fourier transform (FFT) it is clearly visible how friction is relevant for frequency values near the pitch motion frequency (rear end bounce), reducing the gap from real-life data. The pitch acceleration amplitude error between model and real data is reduced by 30%, while front load variation error is reduced

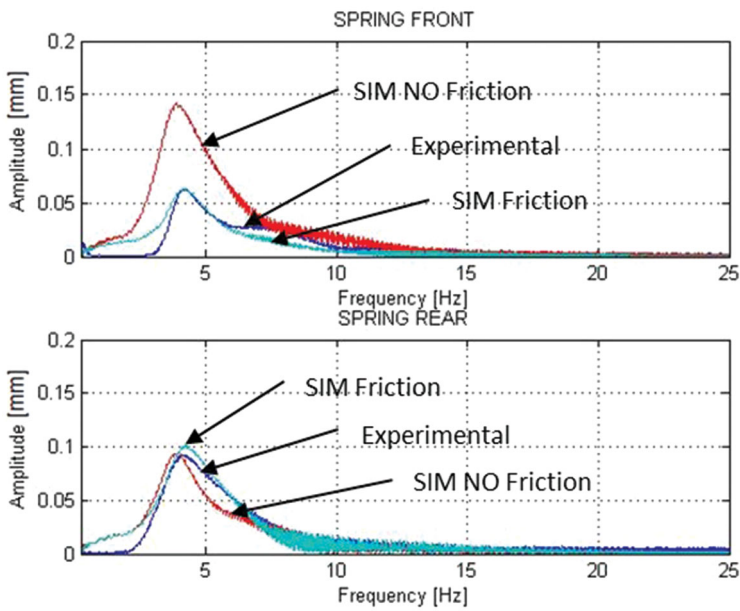


Figure 7. Front and rear spring displacement FFT for the Half Car model.

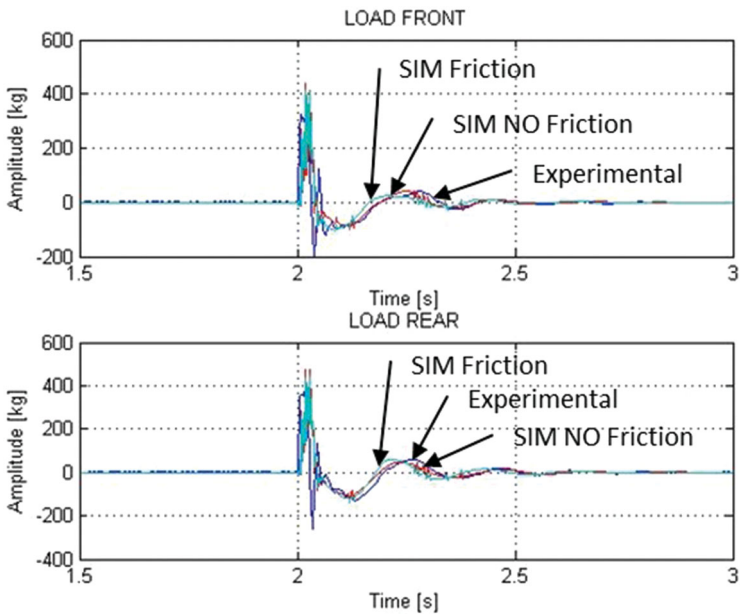


Figure 8. Time domain tyre load variation comparison for the Half Car model.

by 35%. Furthermore, friction acts against sprung and unsprung masses relative motion reducing suspension motion: in fact, spring displacement FFT shows that virtual models with friction better fit experimental data, especially for the front suspension, where friction seems to have a greater influence because of the rear end pitch oscillation.

As regards the time domain analysis, it is worth noticing that the analytical model shows no significant differences with or without friction, faithfully reproducing vehicle response after the vertical step input.

3.4. Results analysis

From the comparison explained above, it is possible to state that friction acts as a further damping contribution to suspension vertical input, increasing vertical and pitch accelerations and tyre load variations response in low frequency range and reducing the ideal spring compression amplitude. Even if this consequence is well known in road vehicle dynamics for high frequencies and low displacement excitations, it is significant to highlight that in race car application the friction effect is relevant also within low frequencies and high displacement amplitude.

Front and rear damping are variables that affect the entire vehicle response during transient manoeuvres, that is vehicle roll at the beginning of a turn and vehicle pitch during accelerations or braking. Obviously, increasing front and rear suspension damping has an effect on the overall vehicle response, as reported in Figure 9 related to the pitch acceleration FFT and the front tyre load variation FFT. The effect is bigger in case of full open damper configuration, where friction is predominant. Instead, when increasing the front dampers coefficient, friction correction is lower but still sensitive. If friction were not included into simulations, the vehicle model accelerations and tyre load variations would show a lower amplitude for excitation frequencies close to bounce and pitch natural frequencies. This means that the virtual model would ignore actual pitch (rear end bounce) motion effects and its consequences on front/rear vehicle ride height variations and aerodynamic balance. Furthermore, it would not take into account the higher front tyre vertical load variations that actually affect front tyre capability to generate lateral forces in that particular frequency range.

Furthermore, friction contribution to the total damping of the system depends on dampers configuration. Simulations in Figure 10 confirm that the friction correction for full closed damper configuration is low, and it affects mostly low relative speed because of the suspension working range. On the contrary, for an open configuration friction, modifies the overall damping force in the entire working velocity range, increasing the mean

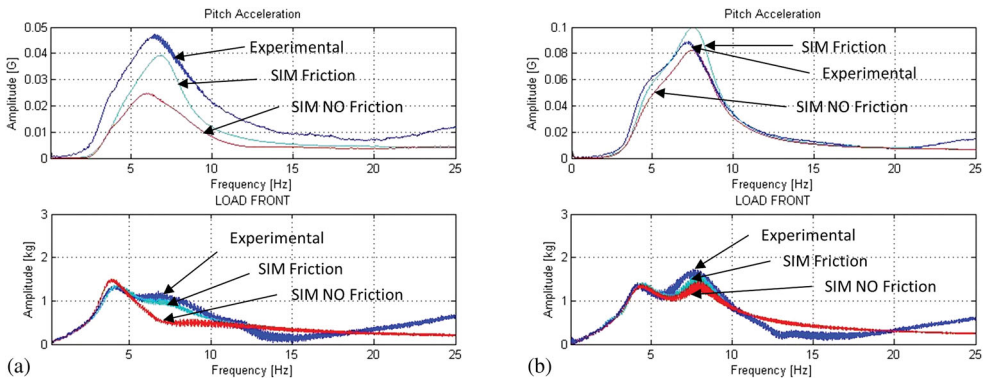


Figure 9. Pitch acceleration FFT and front tyre load FFT for full open (a) and full closed (b) front damper.

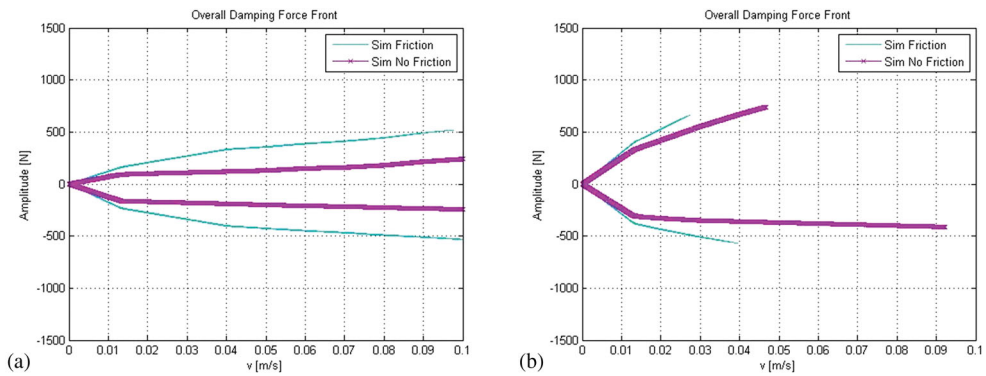


Figure 10. Comparison of front total damping force with full open (a) and full closed (b) front damper configuration. Negative velocity values are bump motion, positive is rebound.

damping coefficient. These differences have to be taken into consideration during suspension setup and cannot be ignored: friction in the overall system increases the nominal damping value depending on the damper actual configuration. As a consequence, it causes a bigger variation in pitch angle and hence in vehicle ride height. These aspects have to be considered in terms of overall aerodynamic balance. Furthermore, friction affects also the vehicle mechanical balance reducing tyre load variation response for bounce frequency excitations while increasing it for a frequency range close to the pitch natural frequency of the car.

4. Conclusion

It is possible to conclude that friction is a relevant factor in vehicle dynamics studies, and in particular in the field of vertical dynamics and in suspension motion analysis. The Four DoF Half Car model clearly improves its capability to represent actual data with the introduction of friction models in front and rear suspension systems, reducing the response error both in frequency domain and in time domain. This effect is particularly evident for the front end of the car and for excitation frequency close to the pitch natural frequency. Regarding the several friction models analysed, the Stribeck model reveals itself as the best description of friction phenomenon. When friction occurs, engineers and mechanics have to deal with it and intervene differently on suspension components. From a practical point of view, it is possible to operate directly on suspension rod ends in order to break-in the system simulating hours of running, and to reduce the friction amount. Considering that friction can be reduced but not eliminated, engineers can also act on dampers modifying their configuration, focusing the attention in particular on vehicle front end low frequency excitations. The rear end motion affects front tyre vertical load and thus the vehicle direction can be compromised. Hence, if setup requires a low damping value, they must take into account that friction increases the overall mean damping coefficient. On the other hand, if setup requires a high damping, they can setup dampers configuration with less click of regulation considering the friction compensation effect.

Future works may further investigate friction effects directly on vehicle handling and driver feedback with the development and simulation of a multibody full vehicle model

which includes friction and its validation by means of real testing on specific proving ground.

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Disclosure statement

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