

# ON THE USE OF THE ADJOINT METHOD TO EVALUATE SENSITIVITIES IN ADSORBED NATURAL GAS STORAGE SYSTEMS

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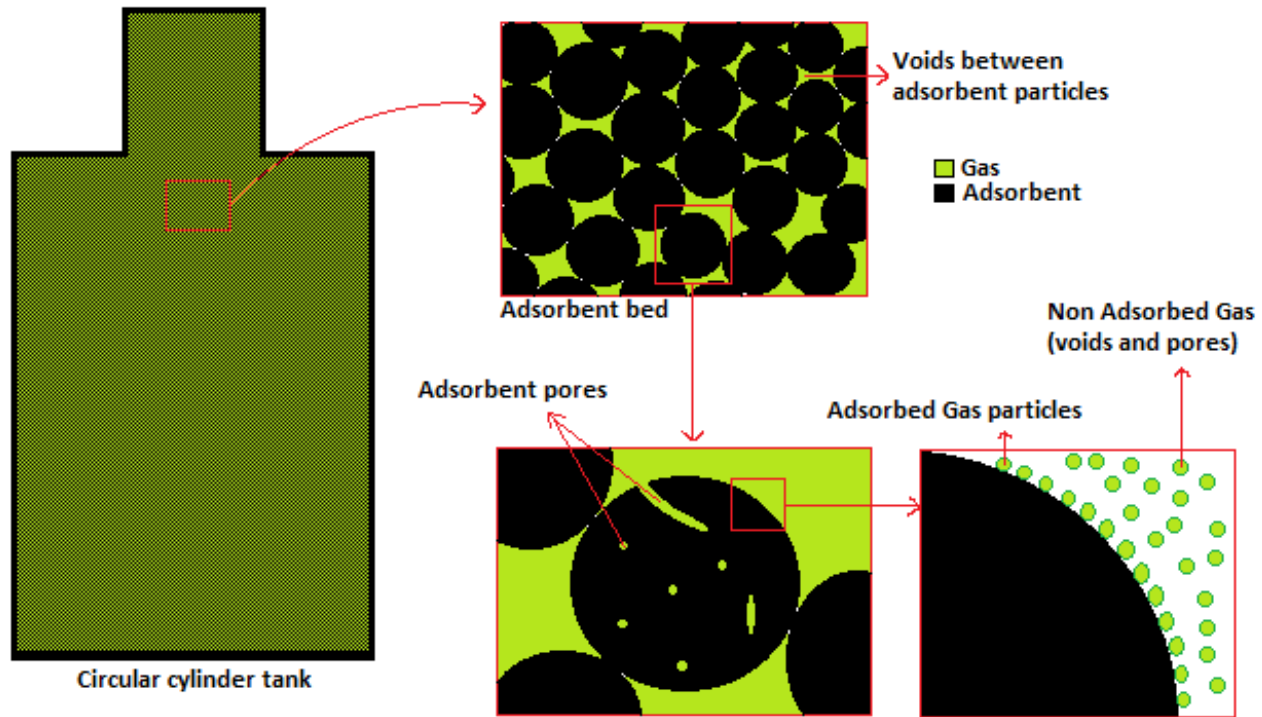
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# Summary

- Technology Description
- Technology Challenges
- Flow Governing Equations
- Adjoint Method
- Results and Conclusions

# Technology Description



# Adjoint Solutions

- Converged Solutions

$$A_{\rho 1} \frac{\partial \sigma}{\partial t} + A_{\rho 2} \frac{\partial \Theta}{\partial t} + A_{\rho 3} \nabla \cdot \psi + A_{\rho 4} \nabla \sigma + A_{\rho 5} \Theta = \frac{\delta R}{\delta \rho}$$

$$A_{v 1} \psi + A_{v 2} \nabla \sigma + A_{v 3} \Theta = \frac{\delta R}{\delta v}$$

$$A_{T 1} \frac{\partial \sigma}{\partial t} + A_{T 2} \frac{\partial \Theta}{\partial t} + A_{T 3} \nabla \cdot \psi + A_{T 4} \nabla \Theta + A_{T 5} \nabla^2 \Theta + A_{T 6} \Theta = \frac{\delta R}{\delta T}$$

# Gradient Validation

- Without Adsorption

$$R = \frac{1}{t} \int_0^t \int_{\Omega} \epsilon_t \frac{\partial \rho}{\partial t} dV dt$$

$$\alpha = G_m$$

$$\frac{\delta R}{\delta \rho} = 0$$

$$\frac{\delta R}{\delta v} = 0$$

$$\frac{\delta R}{\delta T} = 0$$

**Analytical:**

$$\frac{\partial R}{\partial G_m} = 1.267 \cdot 10^{-3}$$

**Finite Difference:**

$$\frac{\partial R}{\partial G_m} = 1.267 \cdot 10^{-3}$$

**Adjoint:**

$$\frac{\partial R}{\partial G_m} = 1.267 \cdot 10^{-3}$$

# Gradient Validation

- Without Adsorption

$$R = \frac{1}{t} \int_0^t \int_{\Omega} \epsilon_t \rho \, dV dt$$

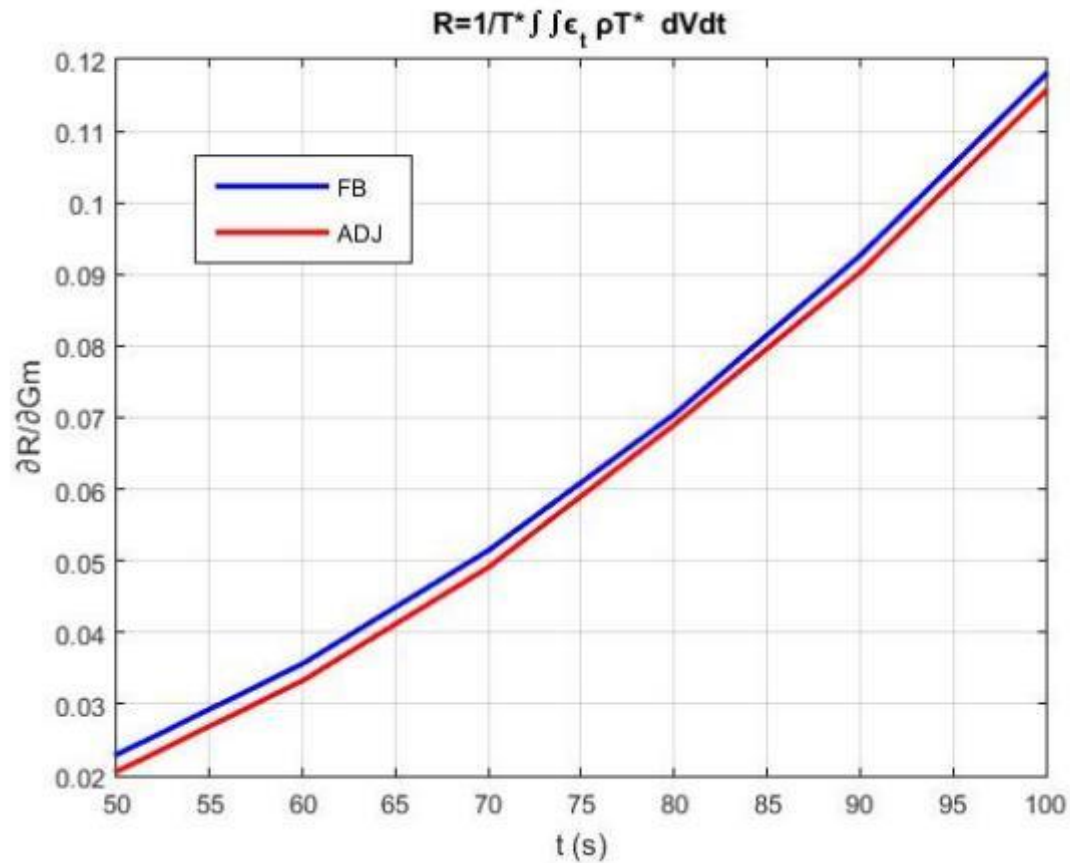
$$\alpha = G_m$$



$$\begin{aligned} \frac{\delta R}{\delta \rho} &= \epsilon_t \\ \frac{\delta R}{\delta v} &= 0 \\ \frac{\delta R}{\delta T} &= 0 \end{aligned}$$

# Gradient Validation

- Without Adsorption



# Gradient Validation

- Without Adsorption

$$R = \frac{1}{t} \int_0^t \frac{1}{V_t} \int p \, dV dt$$

$$\alpha_1 = G_m$$

$$\alpha_2 = Nu$$



$$\frac{\delta R}{\delta \rho} = \frac{T}{V_t}$$

$$\frac{\delta R}{\delta v} = 0$$

$$\frac{\delta R}{\delta T} = \frac{\rho}{V_t}$$



# Gradient Validation

$$\delta^+ = 10 \text{ min} \quad \delta^+ = 10 \text{ min}$$

$$\delta^- = 10 \text{ min} \quad \delta^- = 10 \text{ min}$$

Control Parameters: ( $G_m, Nu$ )	$R = \frac{1}{t} \int_0^t \frac{1}{V_t} \int p \, dV dt$					
	$Nu = 23.6636$ $G_m = 11.150 \frac{kg}{m^2 s}$					
t (s)	$\frac{\partial R}{\partial G_m}$			$\frac{\partial R}{\partial Nu}$		
	BF	ADJ	Dif.	BF	ADJ	Dif.
50	0.39997	0.36935	7.65%	-8.418 e-05	-8.372 e-05	0.6%
60	0.52039	0.50397	3.15%	-1.453 e-04	-1.443 e-04	0.7%
70	0.64813	0.64605	0.32%	-2.289 e-04	-2.272 e-04	0.7%
80	0.78094	0.79405	1.68%	-3.386 e-04	-3.368 e-04	0.6%
90	0.91758	NAN	-	-4.750 e-04	NAN	-
100	1.05730	NAN	-	-6.416 e-04	NAN	-

*Flow = 10 min*  
*Adj = 10 min*

# Gradient Validation

- Without Adsorption

$$R = \frac{1}{t} \int_0^t \frac{1}{V_t} \int T dV dt$$

$$\alpha_1 = G_m$$

$$\alpha_2 = Nu$$



$$\frac{\delta R}{\delta \rho} = 0$$

$$\frac{\delta R}{\delta v} = 0$$

$$\frac{\delta R}{\delta T} = \frac{1}{V_t}$$

# Gradient Validation

$$\delta^+ = 10 \text{ min} \quad \delta^+ = 10 \text{ min}$$

$$\delta^- = 10 \text{ min} \quad \delta^- = 10 \text{ min}$$

Control Parameters: ( $G_m, Nu$ )	$R = \frac{1}{t} \int_0^t \frac{1}{V_t} \int T dV dt$					
	$Nu = 23.6636$ $G_m = 11.150 \frac{kg}{m^2 s}$					
t (s)	$\frac{\partial R}{\partial G_m}$			$\frac{\partial R}{\partial Nu}$		
	BF	ADJ	Dif.	BF	ADJ	Dif.
50	4.020 e-04	1.314 e-03	126%	-8.784 e-06	-8.756 e-06	0.4%
60	5.123 e-04	1.752 e-03	142%	-1.128 e-05	-1.126 e-05	0.4%
70	6.252 e-04	2.200 e-03	151%	-1.411 e-05	-1.406 e-05	0.5%
80	7.384 e-04	2.646 e-03	158%	-1.721 e-05	-1.711 e-05	0.6%
90	8.509 e-04	NAN	-	-2.060 e-05	NAN	-
100	9.618 e-04	NAN	-	-2.424 e-05	NAN	-

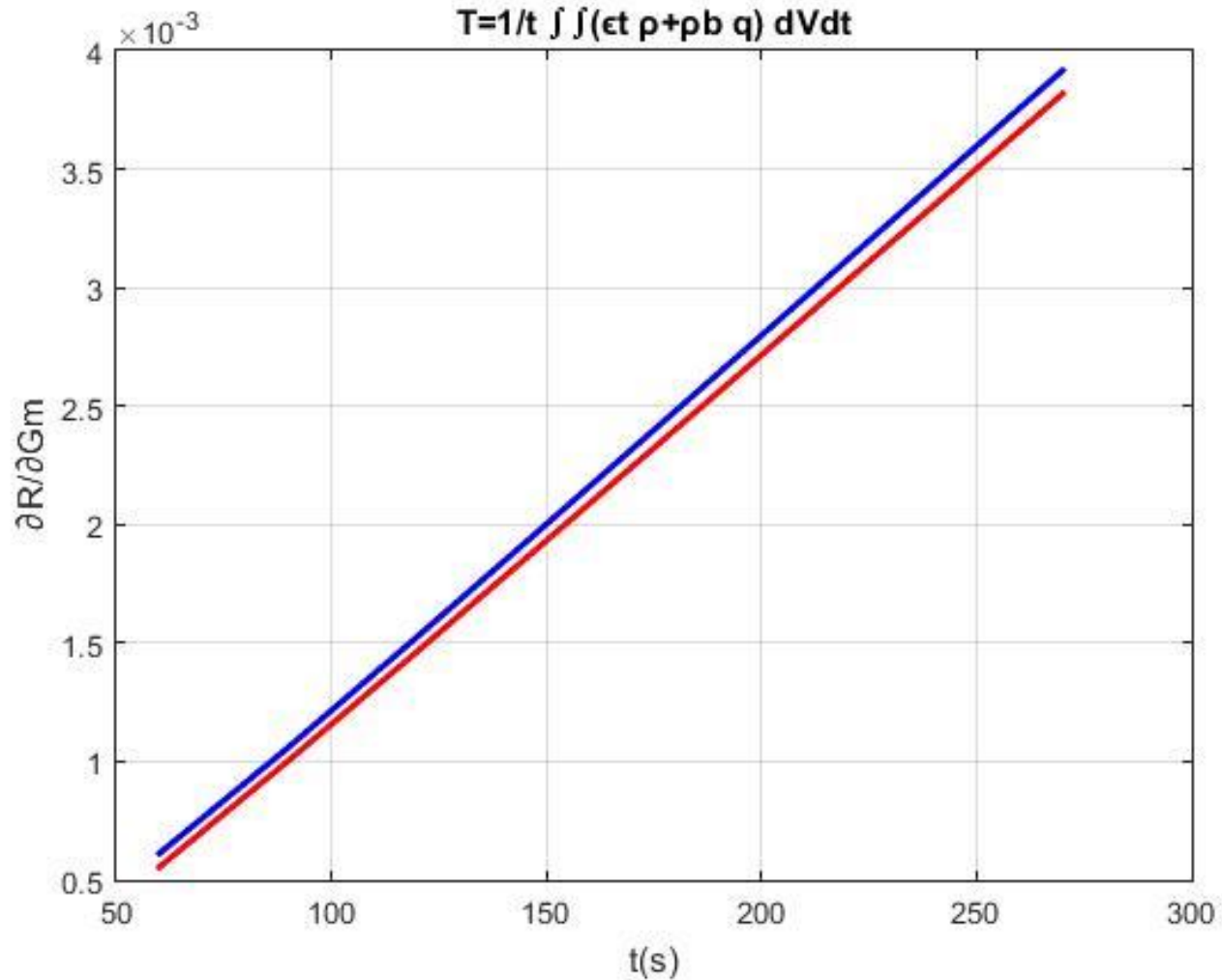
*Flow = 10 min*  
*Adj = 10 min*

# Gradient Validation

- With Adsorption

$$R = \frac{1}{t} \int_0^t \int (\epsilon_t \rho + \rho_b q) dV dt \longrightarrow \begin{aligned} \frac{\delta R}{\delta \rho} &= (\epsilon_t + \rho_b A_\rho) \\ \frac{\delta R}{\delta v} &= 0 \\ \frac{\delta R}{\delta T} &= \rho_b A_T \end{aligned}$$
$$\alpha_1 = G_m$$

# Gradient Validation



# Acomplishments and Next Steps

- Bibliography survey : ✓
- Understanding of the flow physics : ✓
- Development of a flow solver: ✓
- Exploratory Simulations to identify optimizations possibilities: ✓
- Derivation of Adjoint Equations applied in flow through porous media: ✓
- Gradient Validation: **Working on it**



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